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Bose–Einstein condensates and precision measurements

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An ongoing challenge in physics is to make increasingly accurate measurements of physical quantities. Bose–Einstein condensates in atomic gases are ideal candidates for use in precision measurement schemes since they are extremely cold and have laser-like coherence properties. In this paper, we review these two attributes and discuss how they could be exploited to improve the resolution in a range of different measurements.

Keywords: Bose–Einstein condensate; interferometer; squeezing

1. Introduction

In 1925, Einstein predicted that a gas that was cold enough and dense enough would experience a phase transition in which a macroscopic fraction of the atoms come to occupy the ground state of the system (Einstein 1925). That lowest-energy component of the gas is now known as a Bose–Einstein condensate (BEC) and has some fascinating properties. One of the most interesting is that it is an example of a large-scale quantum object. We normally associate quantum effects with microscopic objects only. However, a BEC displays quantum behaviour such as interference and is macroscopic (or at least mesoscopic), in the sense that it contains a large number of particles (up to billions all in the same quantum state) and has a large spatial extent (typically 100 μm).

Since BECs were first predicted, enormous experimental effort has been devoted to creating them in the laboratory. This has focused in recent years on developing techniques for atom cooling and trapping and culminated in 1995 with the first reported observation of a BEC in an atomic gas (Anderson *et al.* 1995). Since then, BECs have been created by numerous research groups around the world and their properties continue to be studied in detail. Attention is now turning to how they can be put to use in a range of new quantum technologies. One area of interest is high precision measurement schemes, for which BECs are particularly well-suited because they are extremely 'cold' and have laser-like coherence properties. In this paper, we will discuss these two distinct properties

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and highlight how each of them enables condensates to be exploited in different measurement schemes.

2. Why are cold atoms interesting for precision measurements?

A notable feature of BECs is that they are extraordinarily cold. By way of example, a condensate of 10^8 atoms of ^{87}Rb in the ground state of a harmonic-trapping potential with a trap frequency of $\omega = 20$ Hz has a diameter of about 100 μm and a corresponding rms velocity of about 7 μm s^{-1}. This is equivalent to a temperature of about 0.2 pK. If we were to switch the trap off, the interactions between the atoms would expand the cloud, leading to higher velocities of around 0.4 mm s^{-1}. However, this still corresponds to an extremely low temperature of approximately 5 nK. In this section, we discuss how this property makes BECs promising candidates for use in precision measurement schemes. We shall focus on how they could be used both to improve time standards and to accurately measure the ratio of Planck's constant to the atomic mass (h/m), which is important in determining the fine structure constant (α).

(*a*) *Measurements of frequencies*

One important area with which the very low temperature of BECs can help is horology. Currently, the best available clocks are atomic fountains, which enable the International System of Units second to be measured with an accuracy greater than 1 part in 10^{15}. These work by cooling caesium atoms to microKelvin temperatures by means of 'optical molasses' generated from three pairs of counter-propagating laser beams in orthogonal directions. The atoms are launched upwards by altering the frequency of the lasers and prepared in the $m_F = 0$ state of one of the hyperfine levels of the electronic ground state. The atoms pass through a microwave cavity (see figure 1) containing a field tuned near to the hyperfine transition frequency for caesium. They continue moving upwards until the Earth's gravity causes them to fall back through the microwave cavity. The two exposures to the microwave field cause some atoms to make the transition between the hyperfine states. The frequency of the microwave field relative to the hyperfine frequency can be inferred by measuring the fraction of atoms in each state after this second transit of the cavity. The uncertainty in the measured frequency scales inversely with the time of flight of the atoms and, so, we want to make this time as long as possible (Ramsey 1956).

Two factors which limit the time of flight are the temperature of the atoms and the effects of gravity. The microwave cavity through which the atoms pass typically has a hole with diameter less than 1 cm and the atoms must pass back through it on their return trip. This means that the atoms should not spread more than 1 cm, which restricts the time of flight for a given temperature. It is clear that colder temperatures allow for longer times and therefore better measurement precision (see figure 1). The other limitations are the effect of gravity and the size of atomic fountains built in the laboratory. For current fountains, which are about one metre high, the return time is approximately one second; for significantly longer times, much taller towers would be needed,

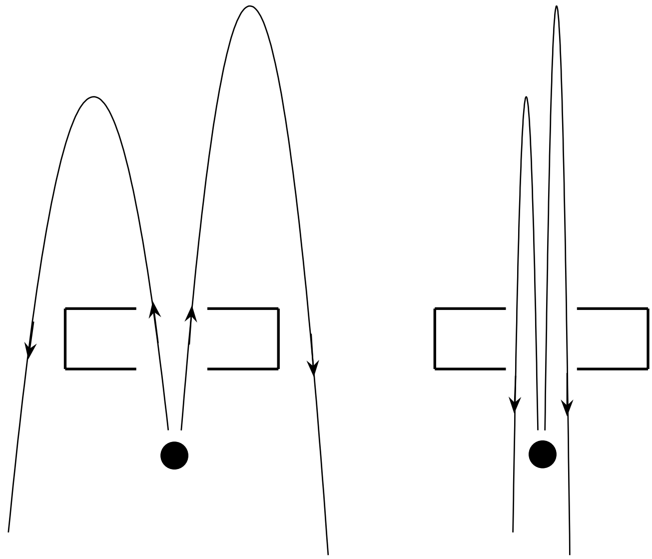

Figure 1. Schematic of an atomic fountain. Laser-cooled atoms are launched upwards through a cavity containing a microwave field. After some time, gravity reverses their direction and they fall back towards the cavity. If the atoms are not cold enough, most will miss the hole in the cavity on the return trip. Colder atoms allow a longer time of flight and, therefore, better measurement resolution.

leading to severe problems with, among other things, the shielding and uniformity of magnetic fields.

It turns out that for earth-bound clocks, the effects of gravity are the limiting factor and laser cooling is sufficient. For a 1 s return time, we require atoms with a velocity spread less than 1 cm s^{-1} so that a substantial fraction passes back through the cavity. Laser-cooled caesium atoms at a temperature of 1 μK achieve this and there is little advantage in cooling them further. A clock in space, by contrast, can have a much longer interaction time than is possible on Earth and therefore will benefit from colder atoms. Raman cooling (Boyer *et al.* 2004) should help but, for very long times, a BEC can provide the really cold temperatures needed. A 100 μm diameter trapped ^{87}Rb condensate that is adiabatically expanded to 1 cm would, upon release, expand at less than 1 μm s^{-1}. Expansion to this or a larger size for a million-atom condensate is needed to reduce the collisional clock shift to a manageable level. In principle, this should allow observation times longer than 1000 s and a significant improvement in the accuracy of clocks.

(*b*) *Measurements of h/m*

Another potential application of the very low temperatures of BECs is in making precise measurements of the ratio h/m, where h is Planck's constant and m is the mass of a particular type of atom. There is a lot of interest in measuring this quantity since it provides another route to determining the fine-structure constant (α). We can see this by writing

$$\alpha^2 = \frac{2R_\infty}{c}\frac{m_\mathrm{p}}{m_\mathrm{e}}\frac{m}{m_\mathrm{p}}\frac{h}{m}. \tag{2.1}$$

This means that with independent measurements of the Rydberg constant, R_∞, the proton–electron mass ratio, $m_\mathrm{p}/m_\mathrm{e}$, and the caesium–proton mass ratio, m/m_p, an accurate determination of h/m can lead to an improved value of α.

Chu and colleagues at Stanford pioneered atom interferometry with laser-cooled atoms to accurately determine h/m by a spectroscopic measurement of the recoil of a caesium atom when it absorbs a photon (Weiss *et al.* 1993, 1994). There are some recent new approaches to this that require atoms with very narrow velocity distributions. One such example (Battesti *et al.* 2004) involves laser-cooling a sample of atoms and then selecting a very narrow (sub-recoil) velocity group from them, $\Delta v \ll \hbar k/m = v_\mathrm{rec}$. This narrow group is then shifted in velocity by an integral number, n, of recoil velocities. The Doppler shift associated with the shifted velocity is given by

$$\Delta\omega = knv_\mathrm{rec} = \frac{nk^2}{2\pi}\frac{h}{m}, \tag{2.2}$$

where k is known from the optical wavelength, which enables h/m to be determined. However, the required sub-recoil velocity selection throws away a large fraction of the atoms. This means that one needs to balance signal against resolution, since a higher velocity resolution results in fewer atoms contributing to the signal. Making use of a BEC would be one way to effectively get the required sub-recoil velocity distribution without losing any of the available atoms.

In another experiment (Gupta *et al.* 2002), a BEC is used to measure h/m, but by the rather different technique of contrast interferometry. In this scheme, a condensate is coherently split into three momentum components, 0 and $\pm 2\hbar k$, by application of a short diffraction pulse of light at time, $t=0$. These components evolve along different trajectories and then a second order Bragg pulse is applied at $t=T$, which reverses the direction of the $\pm 2\hbar k$ components so that all three components combine at $t=2T$ to produce high contrast interference fringes. Each component acquires a phase shift according to its kinetic energy. This gives rise to an atomic density grating whose contrast oscillates with time at an integral multiple of the photon recoil frequency

$$\omega_\mathrm{rec} = \frac{\hbar k^2}{2m}, \tag{2.3}$$

where k is the wavenumber of the photons absorbed by the atoms. Measuring the oscillation of the intensity of the Bragg scattering of light from this oscillating-contrast-grating enables h/m to be determined. The BEC experiment (Gupta *et al.* 2002) had a large systematic error due to mean-field effects—the high BEC density and large scattering cross-section shifts the energy. However, this problem should be able to be overcome if one could first expand the condensate to lower the density sufficiently.

3. Interactions and squeezed atoms

A laser is more than just a narrow-bandwidth light source; it also has a macroscopic occupation of a single mode of the electromagnetic field. A BEC is an analogue of a laser for atoms as it also has a macroscopic number of atoms in a single de Broglie 'atomic mode'. The examples presented so far simply use the remarkably cold nature of the BEC, which is like only using the laser for its narrow bandwidth. However, just as the laser enabled the advent of non-classical states of light such as squeezing, correlated photons and entanglement, BECs enable similar phenomena with atom fields, promising improved measurement capability.

In this section we review recent experiments and proposals for how entanglement may be used in enhanced precision measurement schemes. The key physical process that enables squeezing in condensates is the interaction between atoms due to collisions. These interactions give rise to correlations or entanglements between the atoms, which include squeezing.

One way of number squeezing condensates is to trap them in an optical lattice. This can be achieved by turning on counter-propagating laser beams across a trapped BEC. These lasers form a standing wave and, due to the optical dipole force, the atoms are trapped in the nodes or antinodes of this standing wave, depending on the sign of the detuning of the lasers from the transition frequency. If the optical lattice is turned on sufficiently slowly, the system will remain in the ground state and can be described by the Bose–Hubbard Hamiltonian (Jaksch *et al.* 1998)

$$H = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{V}{2} \sum_i a_i^\dagger a_i^\dagger a_i a_i, \tag{3.1}$$

where a_i is the annihilation operator for an atom at site i and, in the first term, the summation is taken over nearest neighbours. The strength of the tunnelling between sites, J, can be adjusted in experiments by simply raising and lowering the potential barrier. The interaction strength between atoms, V, is, at best, only weakly dependent on the potential but can be controlled by using Feshbach resonances. When the coupling between sites is large compared with the interaction strength, the number of fluctuations at each site are of the order $\sqrt{N}$, where N is the average number of particles per site. As V/J is increased, the atom number fluctuations at each site are progressively reduced (Fisher *et al.* 1989; Jaksch *et al.* 1998), evidence for which was observed in Orzel *et al.* (2001). For sufficiently large values of V/J, a quantum phase transition to the Mott insulator state has been predicted (Jaksch *et al.* 1998) and observed (Greiner *et al.* 2002). In the limit $V/J \to \infty$, each site has precisely the same number of atoms assuming commensurability, that is, the ratio of atoms to sites is an integer.

We can understand this by considering the simple case of the two-well 'lattice' with a total of two atoms in the system (see figure 2). There are two competing processes in this system: on one hand, it is energetically favourable for atoms to hop between the wells and, on the other hand, it is energetically unfavourable for both atoms to occupy the same site due to the additional interaction energy, V, between them. In the strong coupling limit $J \gg V$, the ground state for

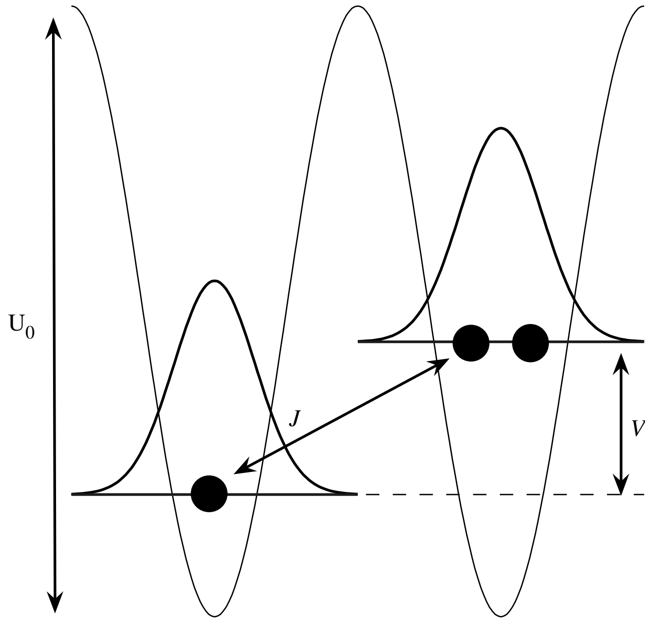

Figure 2. Energy diagram of two atoms in a two-well potential. There are two energies associated with this system: J, which characterizes the tunnelling between the wells and V, which is the energy cost for two atoms to occupy the same well due to interactions between the atoms. By adiabatically increasing the ratio V/J, the system becomes number squeezed.

the system is

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}[\phi_a(1) + \phi_b(1)]\tfrac{1}{\sqrt{2}}[\phi_a(2) + \phi_b(2)], \tag{3.2}$$

where $\phi_j(i)$ is the wave function for atom i on site j. This can be written in terms of the number of atoms at each site as

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}|1\rangle_a|1\rangle_b + \tfrac{1}{2}[|2\rangle_a|0\rangle_b + |0\rangle_a|2\rangle_b], \tag{3.3}$$

where the notation $|i\rangle_j$ represents i atoms at site j. We can see from equation (3.2) that, in this limit, the atoms are not entangled. In the other limit, $V \gg J$, (that is, strong interactions) the ground state is

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}[\phi_a(1)\phi_b(2) + \phi_a(2)\phi_b(1)], \tag{3.4}$$

which can also be written as, $|\psi\rangle = |1\rangle_a|1\rangle_b$.[1] This is the two-well analogue of the Mott insulator state in which each lattice site has exactly the same number of atoms. The number fluctuations have been greatly diminished by the large energy cost, V, of having more than one atom at a site. We can see from equation (3.4) that the two atoms are entangled in this limit because we cannot write the state as a tensor product of the state of each individual particle. This demonstrates how the interactions lead to an entanglement of the atoms.

[1] The entangled, and therefore non-factorizable, state can be written as a product if one uses this basis of the number of atoms at each site.

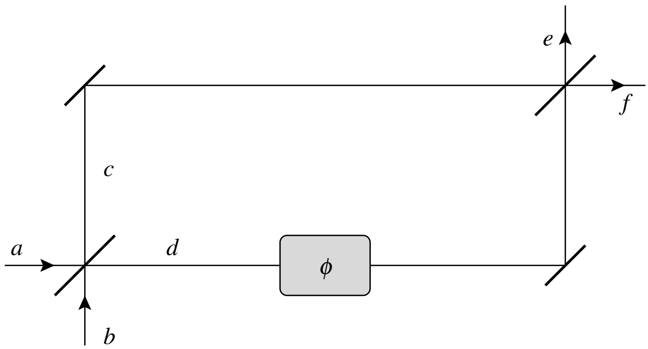

Figure 3. Schematic of a Mach–Zehnder interferometer. Two input modes a and b pass through a 50 : 50 beam splitter giving outputs c and d. Mode d then experiences a phase shift, ϕ, relative to c before the two paths are combined at a second 50 : 50 beam splitter to gives outputs e and f.

The key to creating these Mott insulator states in the laboratory is being able to adiabatically move between the strong coupling regime ($J \gg V$) and the strong interaction regime ($V \gg J$) so that the system remains in the ground state throughout. This has been achieved in experiments with BECs in three-dimensional lattices with more than 10^5 sites (Greiner *et al.* 2002). The experimental success of this scheme has led to significant interest in the possibilities of Mott states as a quantum resource. Various proposals have been put forward for their use, including using them as an array of qubits for quantum computation. In this section, we will focus on how they can be utilised in schemes to accurately measure phase shifts and, for simplicity, will restrict ourselves to the two-well case discussed above.

One possibility is to use a state of the form $|\psi\rangle = |N\rangle_a |N\rangle_b$ as the input to an interferometer (see figure 3). Beam splitters for condensates may be simply realized by allowing tunnelling between the wells for the correct length of time. To illustrate this process, we first consider the effect of passing a two-atom, dual Fock state through a 50 : 50 beam splitter, assuming at this point that the atoms are non-interacting. This gives (Hong *et al.* 1987)

$$|1\rangle_a |1\rangle_b \rightarrow \tfrac{1}{\sqrt{2}}[|2\rangle_c |0\rangle_d + |0\rangle_c |2\rangle_d], \tag{3.5}$$

which has a Schrödinger cat-like form. States of this form are known to be able to give enhanced measurement resolution over states that are not entangled (Huelga *et al.* 1997). We can see this from the form of the state after the phase shift and just before the second beam splitter in figure 3

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}[|2\rangle_c |0\rangle_d + \mathrm{e}^{\mathrm{i}2\phi} |0\rangle_c |2\rangle_d]. \tag{3.6}$$

The phase in this state is enhanced by a factor of 2 over the single particle case: $|1\rangle_a |0\rangle_b \rightarrow (i|1\rangle_c |0\rangle_d + \mathrm{e}^{\mathrm{i}\phi} |0\rangle_c |1\rangle_d)/\sqrt{2}$. However, we should compare the sensitivity of these cases when the same number of atoms is used in each. Even if we allow for two particles to enter the interferometer at port a, the phase resolution that can be achieved in this case is still a factor of $\sqrt{2}$ worse than can be achieved with equation (3.6). This highlights the advantage of using dual Fock state inputs in the interferometer.

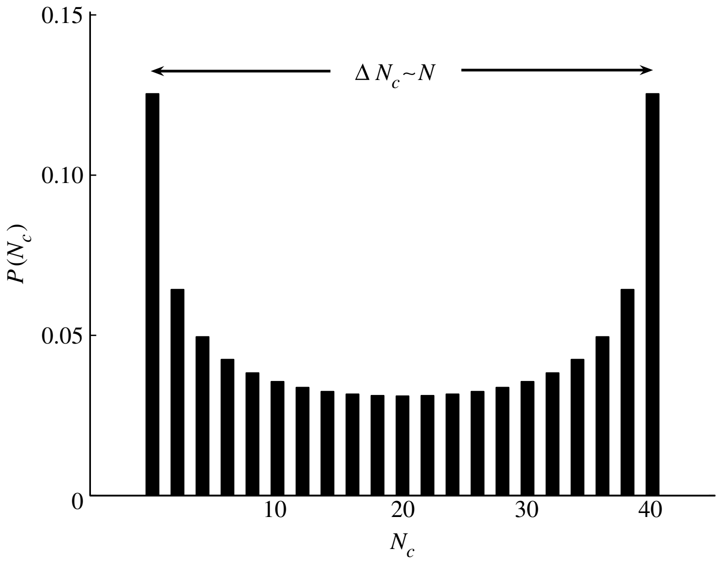

Figure 4. Probability distribution for the number of atoms in mode c, given by equation (3.7).

This result can readily be generalized to multi-particle systems. For N particles at each input port we obtain (Holland & Burnett 1993)

$$|N\rangle_a|N\rangle_b \rightarrow \frac{1}{2^N}\sum_{m=0}^{N}\frac{\sqrt{(2m)!(2N-2m)!}}{m!(N-m)!}|2m\rangle_c|2(N-m)\rangle_d, \tag{3.7}$$

immediately after the first beam splitter. The probability distribution of the number of particles in mode c, $P(N_c)$, for this state is plotted in figure 4 for $N=20$. The most notable feature of this distribution is that the number fluctuations are now large. In fact, the fluctuations are of the same size as the total number of particles in the system, $\Delta N_c \sim N$. This is interesting because it means that the phase fluctuations may be very small. We can see from the number-phase minimum uncertainty relation, $\Delta N_c \Delta\phi \sim 1$, that the phase uncertainty of our state inside the interferometer can scale as $\Delta\phi \sim 1/N$. This is the so-called Heisenberg limit, which is better than the best possible number scaling that can be achieved with uncorrelated particles, for which the result is the standard quantum limit, $\Delta\phi \sim 1/\sqrt{N}$.

This suggests that by using dual Fock states as the input to an interferometer, phase shifts may be able to be measured with Heisenberg limited accuracy (Holland & Burnett 1993). If we were to detect atoms at the output ports e and f, the phase shift would not be encoded on the population difference between the two outputs, $J_z=(e^\dagger e - f^\dagger f)/2$, as is the case in normal interferometry; in fact, $\langle J_z\rangle=0$ in this case. But rather, it can be extracted from a measurement of the variance of this quantity (that is, $(\Delta J_z)^2$) derived from the difference between repeated realizations. The problem with determining $(\Delta J_z)^2$ by directly measuring the number is that it is acutely sensitive to the effects of any deviation from unit detector efficiencies. It has been shown that, to measure the phase shift

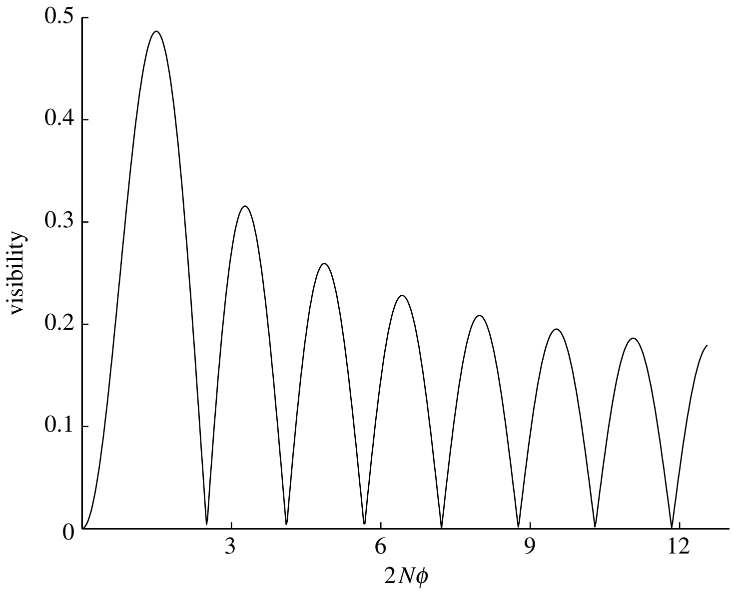

Figure 5. Plot of the visibility as a function of $2N\phi$ of the interference fringes seen when BECs are imaged after undergoing relative phase diffusion for the optimum hold time, t.

with Heisenberg-limited accuracy using this scheme, we would need detectors with efficiencies better than $1-1/N$ (Kim *et al.* 1999). This renders such an approach impractical.

However, it is possible to overcome this problem by disentangling the atoms before measurements are made on them. This can be achieved by passing the state that exits the interferometer back through the Mott transition. By this, we mean that the potential barrier between e and f is adiabatically lowered until the strong coupling regime $J \gg V$ is reached. This process transfers the Heisenberg-limited phase information on to single particle states and therefore it does not matter whether some particles are not detected when the measurement is made (Dunningham *et al.* 2002).

Another approach to making a Heisenberg-limited measurement of the phase shift that is only weakly dependent on the detector efficiencies is to make use of the collapses and revivals of the relative phase between the output modes (Dunningham & Burnett 2004). The rate at which the phase of a state collapses depends on the width of its number distribution (Wright *et al.* 1996). This is because the phase of each number state in a superposition evolves at a different rate in the presence of nonlinearity (due, for example, to atom–atom interactions). The phase diffuses rapidly for a broad distribution, since there is a broad range of phase evolution rates, and diffuses slowly for a narrow distribution. So, by measuring the rate of phase collapse, it is possible to determine $(\Delta J_z)^2$ and, hence, the phase shift, ϕ. In practice, this could be achieved by holding the output state from the interferometer (figure 3) for some time, t, and allowing the relative phase to diffuse. The trapping potentials could then be switched off, releasing the condensates and allowing them to overlap. The rate of collapse could be determined from the visibility of the interference fringes as a function of t over an ensemble average of measurements.

In figure 5, we have plotted an example of how the visibility of the interference fringes varies with $2N\phi$ for a particular hold time, t. This hold time has been chosen to optimize the dependence of the contrast with the phase shift, ϕ (Dunningham & Burnett 2004). We see that the fringes appear and disappear by changing the phase by an amount of order $1/N$, thereby allowing us to resolve phases to the Heisenberg limit. This is a dramatic observable that should be able to be seen in the laboratory and is an exciting demonstration of the great prospects that the coherence properties of BECs bring to the field of metrology.

4. Conclusion

We have highlighted how atomic BECs could be used in a new generation of precision measurement schemes. In §2, we discussed how the very low 'temperatures' (narrow velocity distributions) of BECs could be exploited. One way is to increase the time of flight possible in atomic fountain schemes. This is of particular interest for proposed clocks in space. Another possibility is for the measurement of the ratio h/m or the fine structure constant α. In this case, BECs provide a source of a large number of atoms within a very narrow velocity range. This suggests an improvement over previous techniques that select a narrow velocity range from a source of cold thermal atoms since, in the case of BECs, improved measurement resolution is not obtained at the cost of the number of atoms contributing to the signal.

In addition to their 'coldness', we have shown in §3 how the coherent laser-like properties of BECs can be exploited. By trapping BECs in optical lattices and adjusting the ratio of the tunnelling rate to the strength of interactions between the atoms, the condensates can be number-squeezed. If these squeezed states are used as the input to an interferometer, it is possible to measure phase shifts with improved resolution. In principle, this scheme should enable us to surpass the best possible measurement resolutions that can be obtained from particles that are not entangled and reach the ultimate quantum limit where the phase resolution scales as $1/N$.

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References

Anderson, M. H. *et al.* 1995 *Science* **269**, 198.

Battesti, R. *et al.* 2004 *Phys. Rev. Lett.* **92**, 253 001.

Boyer, V., Lising, L. J., Rolston, S. L. & Phillips, W. D. 2004 *Phys. Rev. A* **70**, 043 405.

Dunningham, J. A. & Burnett, K. 2004 *Phys. Rev. A* **70**, 033 601.

Dunningham, J. A., Burnett, K. & Barnett, Stephen M. 2002 *Phys. Rev. Lett.* **89**, 150 401.

Einstein, A. 1925 *Sitzungber. Preuss. Akad. Wiss.* **1925**, 3.

Fisher, M., Weichman, B., Grinstein, G. & Fisher, D. 1989 *Phys. Rev. B* **40**, 546.

Greiner, M. *et al.* 2002 *Nature* **415**, 39.

Gupta, S., Dieckmann, K., Hadzibabic, Z. & Pritchard, D. E. 2002 *Phys. Rev. Lett.* **89**, 140 401.

Holland, M. J. & Burnett, K. 1993 *Phys. Rev. Lett.* **71**, 1355.

Hong, C. K., Ou, Z. Y. & Mandel, L. 1987 *Phys. Rev. Lett.* **59**, 2044.
Huelga, S. F. *et al.* 1997 *Phys. Rev. Lett.* **79**, 3865.
Jaksch, D. *et al.* 1998 *Phys. Rev. Lett.* **81**, 3108.
Kim, T. *et al.* 1999 *Phys. Rev. A* **60**, 708.
Orzel, C. *et al.* 2001 *Science* **291**, 2386.
Ramsey, N. F. 1956 *Molecular beams.* Oxford: Clarendon Press.
Weiss, D. S., Young, B. C. & Chu, S. 1993 *Phys. Rev. Lett.* **70**, 2706.
Weiss, D. S., Young, B. C. & Chu, S. 1994 *Appl. Phys. B* **59**, 217.
Wright, E. M., Walls, D. F. & Garrison, J. C. 1996 *Phys. Rev. Lett.* **77**, 2158.