Creation of macroscopic superpositions of flow states with Bose-Einstein condensates

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We present a straightforward scheme for creating macroscopic superpositions of different superfluid flow states of Bose-Einstein condensates trapped in optical lattices. This scheme has the great advantage that all the techniques required are within the reach of current experiments. Furthermore, the relative difficulty of creating superpositions scales favorably with the size of the state. This means that this scheme may be well suited to creating superpositions involving large numbers of particles. Such states may have interesting technological applications such as making quantum-limited measurements of angular momentum.

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I. INTRODUCTION

Quantum mechanics allows objects to exist in a coherent superposition of different states. This does not depend on the size of the system and so offers the fascinating prospect of being able to create superpositions of macroscopically distinct states. Multiparticle superposition states have been observed in a number of systems including three photons [1], C_{60} molecules [2], and the internal state of four $^9Be^+$ ions [3]. Experimental signatures of larger-scale quantum phenomena were shown when Rouse *et al.* [4] observed resonant tunneling between two macroscopically distinct states in a superconducting quantum interference device (SQUID). The observed superposition was between states of different flux or opposite currents flowing around a loop. These currents consisted of approximately 10^9 Cooper pairs, meaning tunneling between two macroscopically distinct states had been achieved. Similar systems have also been used to create macroscopic superpositions of these two supercurrent states [5,6].

Bose-Einstein condensates (BEC's) are a promising system for realizing similar results. They are composed of up to 10^9 atoms with a high proportion in the same quantum state and are sufficiently cold to undergo a quantum phase transition from a superfluid to a Mott insulator [7]. There have already been a number of theoretical proposals for producing macroscopic superpositions of BEC's in a range of different setups [8–12].

In this paper, we present a scheme for producing a macroscopic superposition of different superfluid flow states (or equivalently angular quasimomentum states) in a ring of coupled BEC's [12]. This is important because it provides a direct manifestation of quantum mechanics at the macroscopic level in a new system. As discussed by Leggett [13], such states are important for testing the limits of the validity of quantum mechanics. All the steps in our scheme are straightforward and should be achievable with present experiments. The BEC system may also have significant advantages over SQUID's since it is highly controllable: the coupling between condensates and the strength of the interactions between atoms can be tuned over many orders of magnitude. This scheme has the added advantage that the degree of control over the system that is required to create a superposition state scales favorably with the number atoms involved. This suggests that such a scheme may be well suited to creating "large" superpositions. Finally, we discuss how a macroscopic superposition of different superfluid flows may also be of technological interest. One possibility is quantum-limited measurements of angular momentum or, equivalently, ultraprecise gyroscopes [14].

II. THE SCHEME

The system we consider consists of condensed atoms trapped by the optical dipole force in a lattice formed by laser light. In particular, we shall consider three lattice sites in a ring geometry (see Fig. 1). Each of the lattice sites is coupled to its neighbors by quantum-mechanical tunneling through the potential barriers separating them. This setup can be achieved experimentally by trapping condensates in the optical potential created by the diffraction of a laser beam by a liquid crystal spatial light modulator. This modulator allows arbitrary three-dimensional trapping potentials to be achieved, which have the added advantage of being able to be varied smoothly with time. Experiments in Oxford [15] have used this technique to trap BEC's in precisely the configuration that we are interested in—i.e., the three-site ring shown in Fig. 1.

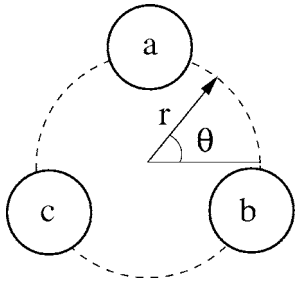

FIG. 1. Diagram of the setup. Three BEC's respectively denoted a, b, and c, are trapped in an optical lattice in a ring geometry. It is convenient to describe this setup in polar coordinates (r, θ).

This system can be described by the Bose-Hubbard Hamiltonian [16]

$$H = -J(a^\dagger b + b^\dagger c + c^\dagger a + \text{H.c.}) + \frac{U}{2}(a^{\dagger 2}a^2 + b^{\dagger 2}b^2 + c^{\dagger 2}c^2), \tag{1}$$

where a, b, and c are the annihilation operators for an atom at the respective lattice sites. The strength of the tunneling between sites, J, can be adjusted in experiments by changing the intensity of the standing wave, thus altering the potential barrier between sites. The interaction strength between atoms, U, is at best only weakly dependent on the potential, but can be controlled by using Feshbach resonances. In the scheme presented here, we will be concerned only with changing the value of J.

Using Bloch's theorem, which describes the wave function of a particle in a periodic potential, we can write the wave function of a Bose-Einstein condensate in polar coordinates as

$$\psi(r, \theta + 2\pi n/3) = e^{i2\pi\ell n/3}\psi(r, \theta), \tag{2}$$

where n is an integer and ℓ is some number to be determined. We can interpret ℓ as being the angular quasimomentum of the mode in units of $\hbar$. Normally when using Bloch's theorem (e.g., to describe electrons moving in a periodic lattice) periodic boundary conditions are put in by hand as an approximation to simplify the analysis. In the geometry considered here, periodic boundary conditions are true physical constraints needed to ensure that the wave function is single valued everywhere. This means that the description of this system using Bloch states is exact. The boundary condition can be written as

$$\psi(r, \theta + 2\pi) = \psi(r, \theta), \tag{3}$$

and we see from Eq. (2) that this requires that ℓ be an integer. Another way of say this is that the angular momentum must be quantized in units of $\hbar$. Moreover, we see from Eq. (2) that changing ℓ by an integer multiple of 3 does not change the wave function. This means that a complete basis is formed by restricting ℓ to the values $\ell \in \{0, \pm 1\}$. The annihilation operators for this complete quasimomentum basis are

$$\alpha = \frac{1}{\sqrt{3}}(a + b + c), \tag{4}$$

$$\beta = \frac{1}{\sqrt{3}}(a + e^{i2\pi/3}b + e^{i4\pi/3}c), \tag{5}$$

$$\gamma = \frac{1}{\sqrt{3}}(a + e^{-i2\pi/3}b + e^{-i4\pi/3}c). \tag{6}$$

Let us now consider the details of the process for creating the superposition state. The great advantage of the scheme presented here is that it is straightforward to carry out and within reach of present experiments. Macroscopic superpositions can be created simply by starting with the ground state in the lattice when the barriers are low, rapidly increasing the intensity of the optical lattice, and then waiting for a certain time before rapidly decreasing the intensity of the optical lattice again.

For simplicity we begin by considering a system of only three atoms [17] (we will consider larger numbers later in the paper). We want to create a state of the form

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|3,0,0\rangle_{\alpha\beta\gamma} + |0,3,0\rangle_{\alpha\beta\gamma} + |0,0,3\rangle_{\alpha\beta\gamma}), \tag{7}$$

i.e., a superposition of all the atoms having no angular momentum and all having one unit of angular momentum clockwise and all having one unit of angular momentum anticlockwise.

The Hamiltonian for the condensate in the lattice when the potential barriers are sufficiently low that the energy associated with tunneling dominates the energy associated with interactions, $J \gg U$, is given approximately by

$$H = -J(a^\dagger b + b^\dagger c + c^\dagger a + \text{H.c.}) = -J(2\alpha^\dagger\alpha - \beta^\dagger\beta - \gamma^\dagger\gamma). \tag{8}$$

This is a very good approximation for modest numbers of atoms and can be further improved using Feshbach resonances to reduce the interaction strength. The effects of any residual interactions may be treated using perturbation theory and will form the subject of future work. The eigenstates of the Hamiltonian (8) are the quasimomentum modes (4)–(6) and the ground state is when all the atoms are in mode α (i.e., the zero-quasimomentum Bloch state). For three atoms, this can be written in the basis of the number of atoms per lattice site as

$$\begin{aligned}|\psi\rangle = \frac{1}{9\sqrt{2}}(a^\dagger + b^\dagger + c^\dagger)^3|0,0,0\rangle = \frac{1}{3\sqrt{3}}(|3,0,0\rangle + |0,3,0\rangle \\ + |0,0,3\rangle) + \frac{\sqrt{2}}{3}|1,1,1\rangle + \frac{1}{3}(|1,2,0\rangle + |2,1,0\rangle + |1,0,2\rangle \\ + |2,0,1\rangle + |0,1,2\rangle + |0,2,1\rangle).\end{aligned} \tag{9}$$

Figure 2 shows the number distribution of atoms in the lattice sites, $P(N_a, N_b) = |\langle N_a, N_b, 3 - N_a - N_b|\psi\rangle|^2$, for this ground state. As we might expect, we see that it is relatively unlikely to find all the atoms in a single lattice site, but more likely to find them equally distributed between sites.

Our claim is that, if we start with a state of the form of Eq. (9), we can create a superposition state of the form of Eq. (7) by first rapidly increasing the intensity of the optical lattice, then waiting for a certain time, and finally rapidly decreasing the light intensity again. We now study the details of this process.

The first step is to rapidly increase the intensity of the trapping laser light, thereby increasing the height of the potential barriers between the sites and so decreasing the coupling, J. We want to do this in such a way that we "lock in" the number distributions in the three sites, but do not excite any higher-lying vibrational states in each site. Such a condition requires that the rate at which the intensity is increased be much faster than the rate of tunneling between lattice sites, but still adiabatic with respect to the spacing of vibra-

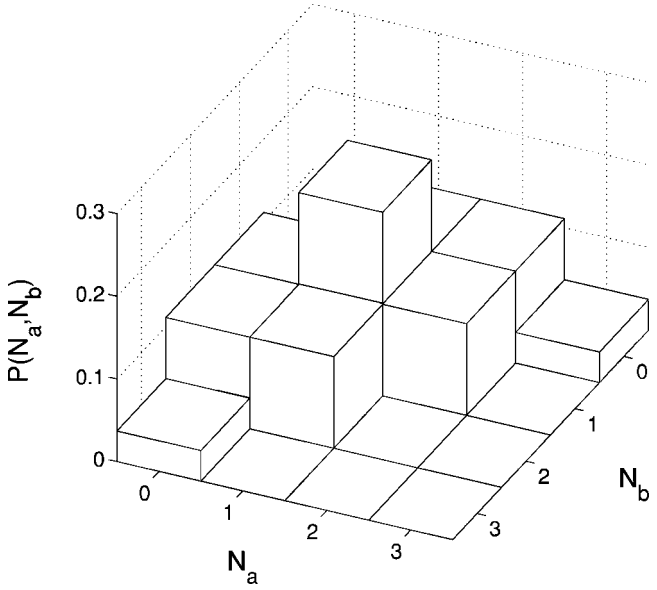

FIG. 2. Probability distribution of the number of atoms in lattice sites a and b for the initial ground state of the system—i.e., all atoms in the α quasimomentum mode—for $N=3$.

tional levels at each site. It turns out that the time scales for these two processes are well separated. Experiments have been carried out that rapidly raise the barriers between trapped BEC's within a time of 50 μs and satisfy precisely these two conditions [18]. If the coupling between the sites is decreased sufficiently by the increase in intensity that the coupling energy is small compared with the energy due to the interactions between atoms, the Hamiltonian can be written approximately as

$$H = \frac{U}{2}(a^{\dagger 2}a^2 + b^{\dagger 2}b^2 + c^{\dagger 2}c^2). \tag{10}$$

It turns out that this is an excellent approximation as the energy associated with the coupling is negligible when the intensity has been increased in experiments [18]. As before, any corrections due to a small residual coupling could be accounted for using perturbation theory. Immediately after the barriers have been raised, the state is given by Eq. (9) since it has not had time to evolve. If we now allow this state to evolve under the influence of the Hamiltonian (10), after time t, the state is

$$|\psi\rangle_2 = \frac{1}{3\sqrt{3}}e^{i3Ut}(|3,0,0\rangle + |0,3,0\rangle + |0,0,3\rangle) + \frac{\sqrt{2}}{3}|1,1,1\rangle + \frac{1}{3}e^{iUt}(|1,2,0\rangle + |2,1,0\rangle + |1,0,2\rangle + |2,0,1\rangle + |0,1,2\rangle + |0,2,1\rangle). \tag{11}$$

We now want to see whether this evolves to a state of the form of Eq. (7). For this to happen, we need the probability for all three atoms to be found in mode α, $P_\alpha(3)$, to be one-third and similarly for the probabilities for modes β, $P_\beta(3)$ and γ, $P_\gamma(3)$. Calculating these probabilities directly we get

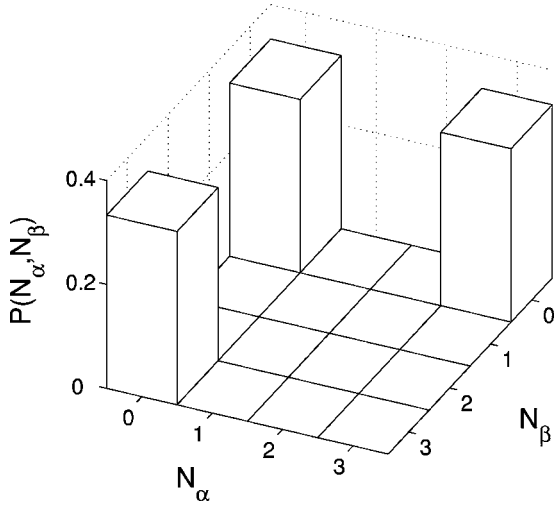

FIG. 3. Final number distribution of atoms in the α and β modes after the state-creation process when $N=3$.

$$P_\alpha(3) = \frac{1}{3^4}[41 + 24\cos(Ut) + 12\cos(2Ut) + 4\cos(3Ut)], \tag{12}$$

$$P_\beta(3) = P_\gamma(3) = \frac{1}{3^4}[41 - 12\cos(Ut) - 6\cos(2Ut) + 4\cos(3Ut)]. \tag{13}$$

If we pick the evolution time to be $t=2\pi/(3U)$, we get $P_\alpha(3)=P_\beta(3)=P_\gamma(3)=1/3$. This is precisely what we want: an equal superposition of all the atoms in α, all in β, and all in γ. A plot of the full number distribution in the quasimomentum basis is shown in Fig. 3 and confirms that a state of the form of Eq. (7) has been created.

The last step is to rapidly lower the potential barriers again. As before, this is done much faster than the rate of tunneling between the potential wells, but slowly with respect to the spacing between vibrational levels within each well. This leaves us with the state (7) [19] evolving due to the Hamiltonian (8). Since this Hamiltonian has no terms that couple different quasimomentum modes, the population in each does not change with time and so the form of the state is preserved. Of course, this is strictly true only if we can ignore the nonlinear interaction terms, as discussed above. Finally, we should note that the relative phases between the terms of the state (7) change with time since the different quasimomentum modes have different energies; i.e., the nonrotating mode α has a lower energy than the rotating modes β and γ.

III. LARGER NUMBERS

We have now demonstrated how the state-creation process works for three atoms. In order for this to be a truly powerful technique, we need to demonstrate that it also works for larger numbers of particles, $N>3$, to create states of the form

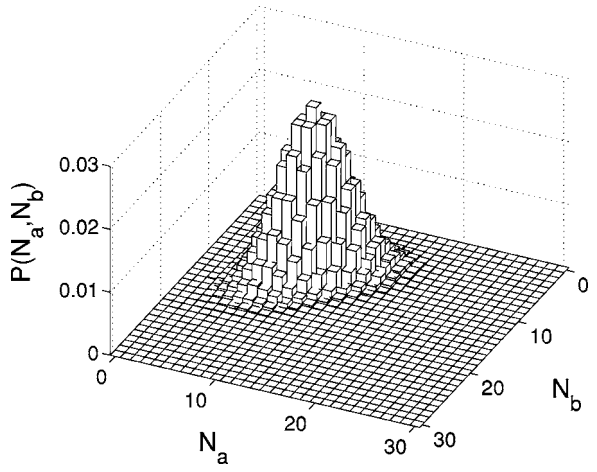

FIG. 4. Probability distribution of the number of atoms in lattice sites a and b for the initial ground state of the system—i.e., all atoms in the α quasimomentum mode—for $N=30$.

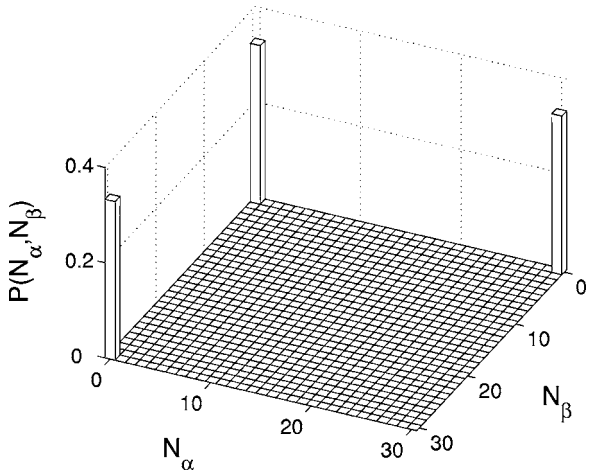

FIG. 5. Final number distribution of atoms in the α and β modes after the state-creation process when $N=30$.

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|N,0,0\rangle_{\alpha\beta\gamma} + |0,N,0\rangle_{\alpha\beta\gamma} + |0,0,N\rangle_{\alpha\beta\gamma}). \quad (14)$$

For simplicity, let us suppose that the total number of atoms in the system is a multiple of three—i.e., $N=3n$, where n is an integer. The initial ground state of the condensed atoms trapped in the lattice when the energy of the coupling term dominates, $J \gg U$, is

$$|\psi\rangle = \frac{1}{\sqrt{3^N N!}}(a^\dagger + b^\dagger + c^\dagger)^N |0,0,0\rangle = \frac{1}{\sqrt{3^N}} \sum_{p=0}^{N} \sum_{q=0}^{N-p} \sqrt{\frac{N!}{p!q!(N-p-q)!}} |p,q,N-p-q\rangle. \quad (15)$$

The probability distribution of the number of atoms in lattice sites a and b is plotted in Fig. 4 for $N=30$.

We now follow exactly the same procedure as for three atoms: rapidly raise the potential barriers, hold for time $t = 2\pi/(3U)$, and then rapidly reduce the barriers to their initial height. It is straightforward to calculate the probabilities for all the atoms to be in a single quasimomentum mode and, as before, we get $P_\alpha(N) = P_\beta(N) = P_\gamma(N) = 1/3$. The full number distribution is shown in Fig. 5 for $N=30$ and confirms that a state of the form (14) has been created. This is a great result—it means that we do not need to know the number of atoms in the initial state. As long as the total number of atoms is a multiple of three, we get Eq. (14).

We would now like to consider what happens when $N \neq 3n$. To achieve this, it is helpful to define a measure C of how similar our final state is to Eq. (14). For this, we choose

$$C = 3[P_\alpha(N)P_\beta(N)P_\gamma(N)]^{1/3}. \quad (16)$$

We choose this quantity as our measure rather than the (perhaps more obvious) fidelity because we are not interested in the phases between the terms, but only in whether we have a macroscopic superposition state. It is possible to have a state that is an equally weighted superposition of all the atoms being in each of the three quasimomentum states and, with carefully chosen phases, is orthogonal to Eq. (14); i.e., the fidelity of this state is zero. This, however, is still a state of the form we want and so C is a better choice for a measure since it is insensitive to the relative phases between the terms. C can take values ranging from 0, when the state is very different from Eq. (14), up to 1, when the state has the same form as Eq. (14). In Fig. 6, we have plotted the value of C for the final state (created by the procedure outlined above) for a range of values of N. We see that we get a state of the form of Eq. (14) if N is a multiple of 3, but not otherwise [20]. This, however, should not overly concern us. For one, this is a very simple procedure for creating large superposition states. The straightforward implementation of this scheme far outweighs the fact that it only works for one in three trials, which is certainly not a prohibitively low success rate for usefulness in a range of schemes. Second, in certain measurement schemes that require a state of the form of Eq. (7), it is possible to post-select trials where the state

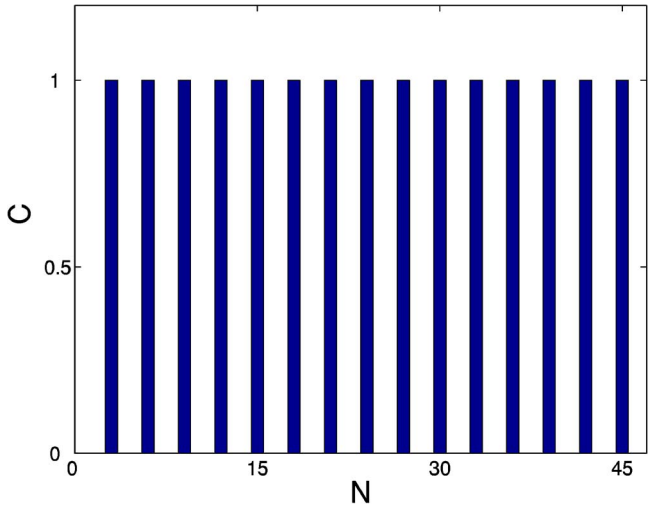

FIG. 6. (Color online) Plot of C defined by Eq. (16) as a function of the total number of atoms in the system, N. A state of the form of Eq. (14) is created (i.e., $C=1$) only if N is a multiple of 3.

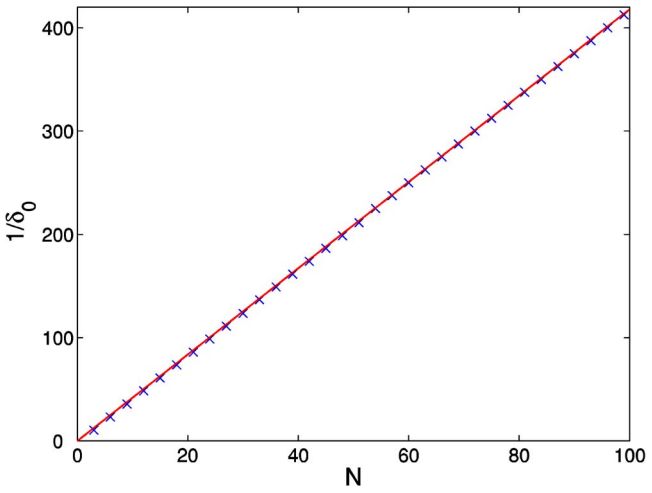

FIG. 7. (Color online) Plot of $1/\delta_0$ as a function of N, where δ_0 is the maximum value of the error in the timing of the nonlinear evolution, δ, that allows $C \geqslant 0.9$. The crosses are numerically calculated data points for values of N that are multiples of 3. The solid curve is a line of best fit intended to find the scaling. The solid curve should not be interpreted as an interpolation between points, since the state-creation process only works when N is a multiple of 3.

was successfully generated and discard the unsuccessful trials. Of course, the superposition is destroyed by the measurement, but post-selection allows us to retain measurement data only from trials that involved the desired state. An example of such a procedure is discussed in Sec. V.

IV. DISCUSSION

One of the great advantages of this scheme over a procedure we previously proposed [12] is that, instead of having to control the rate of rotation of the lattice to a high degree of precision, we need only control the time that the system is allowed to evolve when the potential barriers are high. This should be much easier to achieve in practice. In this section, we investigate how sensitive the state-creation process is to the accuracy of this timing.

As our benchmark we will take a value of $C=0.9$. This corresponds (in the case of the state being symmetric in α, β, and γ) to 90% of the atoms being in a state of the form of Eq. (14). We would now like to investigate how accurately we need to control the time of the nonlinear evolution in order to achieve a value of $C \geqslant 0.9$ for the final state. If we take the time of the nonlinear evolution to be $t=(1+\delta)2\pi/(3U)$, where δ is the error in the timing, we would like to find the maximum value of δ such that $C \geqslant 0.9$ as a function of N. We denote this maximum value as δ_0.

A plot of these results is shown in Fig. 7. The crosses show the points for total numbers of atoms that are a multiple of 3 and the solid curve is a best-fit line. The approximate scaling from this fit is

$$\delta_0 \approx 0.24/N. \tag{17}$$

This means that the error that can be tolerated in the timing scales as $\Delta t \sim 1/UN$. We note that this result is consistent with the calculation in the previous section for three particles. Eqs. (12) and (13), the most rapidly varying terms are proportional to $\cos(3Ut)$, which means that the error in t scales as $\Delta t \sim 1/(3U)$.

Here we see the great advantage of this scheme: the accuracy with which we need to control the timing scales as $1/N$. This is much more favorable than the exponential dependence on N that the rate of rotation of the lattice had to be controlled in a previous scheme [12]. This means that the present scheme has a relatively low premium on creating macroscopic superposition states with large numbers and so is particularly well suited to this purpose. This is good news since the advantages of superposition states in quantum schemes and investigating fundamental issues of physics are more fully exploited the larger the superposition is.

The fact that the nonlinear evolution needs to be controlled more accurately the larger the superposition is will ultimately limit the size of states that can be created by this process. However, this fact could also be turned to an advantage. For example, we could use this scheme to measure the value of U accurately—something that is difficult to achieve experimentally. One way this could be achieved is to carry out the procedure outlined above for different values of t and then make measurements on the final state. Whenever Ut is a multiple of $2\pi/3$, a state of the from of Eq. (14) will be created when N is a multiple of 3. This means that one-third of the time we would find all the atoms in the same angular momentum mode—a highly unlikely event if a state of the form of Eq. (14) had not been created. This, then, provides a clear means of calibrating Ut to within $1/N$ and, since time can be measured extremely accurately, would enable a precise determination of U.

It is well known that macroscopic superposition states are fragile to the effects of loss and that this sensitivity increases as the superposition gets larger. In practice, this will limit the size of superposition states that can be created in the laboratory. This restriction, however, is not fundamental and, by carefully eliminating sources of decoherence, superpostions of increasing size should be able to be realized. Experiments have already successfully observed superpositions of large numbers of particles in C_{60} molecules [2] and supercurrents [5,6].

V. READOUT AND INTERFEROMETRY

One interesting application of macroscopic superpositions of flow states is that they may allow for quantum-limited measurements of angular momentum—i.e., ultraprecise gyroscopes [12,14]. To achieve this, we would like to carry out a form of interferometry using macroscopic superposition states. A similar idea makes use of "quantum beam splitters" to accurately measure phase shifts in optical and double-well BEC systems [21,22].

The basic idea is to create a macroscopic superposition of superfluid flows using the procedure outlined above. The state is then held for some time Δt, and any rotation of the lattice during this time will shift the energy levels of the different angular quasimomenta and so encode phases on the state. The state-creation procedure is then repeated, in anal-

ogy with the second beam splitter in a Mach-Zehnder interferometer, and the number of atoms in each angular momentum mode is measured. This should enable the rotation of the lattice to be determined accurately.

In the present discussion, we will limit ourselves to the case where the total number of atoms is a multiple of 3. In this case, it can be shown that the transformation of the state by the state-creation process is

$$\begin{pmatrix} |N,0,0\rangle_{\alpha\beta\gamma} \\ |0,N,0\rangle_{\alpha\beta\gamma} \\ |0,0,N\rangle_{\alpha\beta\gamma} \end{pmatrix} \rightarrow U \begin{pmatrix} |N,0,0\rangle_{\alpha\beta\gamma} \\ |0,N,0\rangle_{\alpha\beta\gamma} \\ |0,0,N\rangle_{\alpha\beta\gamma} \end{pmatrix}, \tag{18}$$

where U is the unitary transformation,

$$U = \frac{1}{\sqrt{3}} \begin{pmatrix} e^{-i2\pi/3} & 1 & 1 \\ 1 & e^{-i2\pi/3} & 1 \\ 1 & 1 & e^{-i2\pi/3} \end{pmatrix}. \tag{19}$$

It is straightforward to show that $U^3=1$. This means that, since that the state-creation process corresponds to the operator U, we pick the second operation (corresponding to the second "beam splitter") to be U^2. This ensures that the second operation is the inverse of the first. Experimentally, this could be achieved simply by allowing the state to evolve twice as long with the nonlinear interactions—i.e., $t=4\pi/(3U)$ [23].

We now need to consider how rotations of the lattice shift the energy of α, β, and γ. For small changes in the angular momentum, ΔL, we can write

$$\Delta E \approx \frac{\partial E}{\partial L}\Delta L = \xi L, \tag{20}$$

where ξ is given by L/I and I is the moment of inertia of the atoms. We saw earlier that the angular momenta of α, β, and γ are 0, $\hbar$, and $-\hbar$, respectively. This means that the respective energy shifts of these modes are 0, $\xi\hbar$, and $-\xi\hbar$ and the Hamiltonian describing the system at this stage can be modified from Eq. (8) to give

$$H = \hbar[-2J\alpha^\dagger\alpha + (J+\xi)\beta^\dagger\beta + (J-\xi)\gamma^\dagger\gamma]. \tag{21}$$

The matrix describing the phase shift of each mode due to evolution with this Hamiltonian for time Δt, $e^{iH\Delta t/\hbar}$, can be written in the basis $\{|N,0,0\rangle_{\alpha\beta\gamma}, |0,N,0\rangle_{\alpha\beta\gamma}, |0,0,N\rangle_{\alpha\beta\gamma}\}$ as

$$Q = \begin{pmatrix} e^{-i3NJ\Delta t} & 0 & 0 \\ 0 & e^{iN\xi\Delta t} & 0 \\ 0 & 0 & e^{-iN\xi\Delta t} \end{pmatrix}, \tag{22}$$

where an overall phase has been neglected.

Before discussing the results, let us first summarize the interferometry scheme. We first create a superposition state by the process described earlier and then hold it for some time, Δt. It is during this time that any slight changes in the rotation of the lattice are detected. Finally, we repeat the state-creation process, but this time for double the duration of the nonlinear interaction, and detect which angular momentum modes the atoms of the final state are in. Even though we have two state-creation processes in this scheme, the overall chance of success in any given trial is still one-third. This is because either both processes are successful or both are not depending on whether the total number of particles in the system is a multiple of 3; i.e., the successes of the two processes are not independent. Furthermore, we can throw away any unsuccessful trials since we will know that it has been unsuccessful when we measure it if all the atoms are not found in the same angular momentum mode α, β, or γ. This means that the restriction that the total number of atoms must be a multiple of 3 should not overly concern us.

The final state created by this scheme, if we begin with the ground state of the system (i.e., all the atoms in the nonrotating α mode), is $|\psi\rangle_{\text{final}} = U^2QU|N,0,0\rangle_{\alpha\beta\gamma}$. When $\xi\Delta t=0$—i.e., there is no rotation of the lattice or the time over which the rotation is applied is vanishingly small—we get $Q=1$ and the final state is the same as the initial state, as it should be. The probabilities for finding all the atoms in α, β, and γ, respectively, are

$$P_\alpha = \frac{1}{9}[1 + 4\cos^2(N\xi\Delta t) + 4\cos(N\xi\Delta t)\cos(3NJ\Delta t)], \tag{23}$$

$$P_\beta = \frac{1}{9}[1 + 4\cos^2(N\xi\Delta t - 2\pi/3) + 4\cos(N\xi\Delta t - 2\pi/3)\cos(3NJ\Delta t)], \tag{24}$$

$$P_\gamma = \frac{1}{9}[1 + 4\cos^2(N\xi\Delta t + 2\pi/3) + 4\cos(N\xi\Delta t + 2\pi/3)\cos(3NJ\Delta t)], \tag{25}$$

and $P_\alpha + P_\beta + P_\gamma = 1$, as it should when N is a multiple of 3.

We see that the frequency of oscillation of the probabilities (23)–(25) scales as N. So by changing $\xi\Delta t = L\Delta t/I$ by an amount of the order of $1/N$ all the atoms can be transferred from one angular momentum mode to another. This is a dramatic observable that should be able to be measured experimentally, thereby enabling measurements of angular momentum with a precision that scales as $1/N$. This is the same scaling as the fundamental quantum limit (Heisenberg limit) and, since N can be large for macroscopic superpositions, opens up the exciting possibility of achieving ultraprecise gyroscopes.

VI. CONCLUSION

We have presented a detailed scheme for creating macroscopic superpositions of different superfluid flow states in an optical lattice. Such states are of fundamental interest in exploring the boundary between the regimes where quantum and classical physics are valid. The scheme we have proposed has some pleasing features. BEC's in optical lattices present a system free from impurities that is highly controllable—the coupling between sites and the interactions between atoms can be adjusted over several orders of magnitude. Furthermore, this scheme is very simple and well within reach of present experiments. Macroscopic superposi-

tion states can be created simply by rapidly switching the intensity of the laser beams that form the optical lattice. This scheme has the great advantage that there is a relatively low premium on creating superpositions with large numbers of atoms. The timing of the nonlinear interaction needs to be controlled to an accuracy that scales as $1/N$, which is much more favorable than the degree of control required in other schemes. This suggests that this scheme may present a viable route to achieving macroscopic superpositions of superfluid flow states in the laboratory. Along with enabling detailed studies of quantum mechanics, these states may have interesting technological applications including ultraprecise quantum-limited gyroscopes.

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