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GENERAL PROBLEMS AND DIRECTIONS

Observing Superpositions of Different Number States[1]

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Abstract—The formal structure of quantum mechanics allows systems to exist in a superposition of different states. A curious feature is that while some superpositions, such as position or momentum, are routinely observed, others such as mass or charge are not. This anomaly is at least partly due to the fact that it is much easier to carry out interference experiments for certain observables than others. Here we focus particularly on the superposition of two different number states for photons or bosonic atoms. This can equally well be thought of as a superposition of different masses in the case of atoms. We present different interferometry schemes that should, at least in principle, allow us to observe interference between these states. This would provide evidence of a superposition that we would normally consider to be prevented by superselection rules.

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INTRODUCTION

Physicists have got used to the idea that quantum states can exist in superpositions, strange as they seem. We have become familiar, for example, with particles being in "two places at once" or being in a superposition of different polarisations in EPR experiments that test local realism. Although certain observables seem to lend themselves to superpositions much more readily than others, there is nothing in the formalism of quantum mechanics that makes this distinction. Instead the idea of superselection rules [1] was put forward to explain why we do not observe certain superpositions in practice.

The fundamental basis for superselection rules is not clear. Indeed Aharonov and Susskind [2] challenged the need for them at all with a thought experiment to observe coherent superpositions of different charge states. Their work showed that it is important that there is an appropriate reference frame in order to observe the coherence [2–7]. For some quantities like position or polarisation, reference frames are readily available, which means that their superpositions are commonly observed. But, for other quantities like mass or charge, the appropriate reference frames are much less obvious and so it is much harder to observe a superposition. The problem of observing coherent superpositions seems to be reduced to the problem of finding a suitable reference frame.

Other authors have extended the ideas of Aharonov and Susskind and applied them to different systems [3–5, 8]. Recent work, for example, has considered whether it might be possible to observe interference fringes that are due to a superposition of an atom and a molecule [6, 7], i.e., two states with different mass. Here we give an overview of the problem and consider different ways in which we might be able to observe superpositions of different Fock states. There has been a lot of interest lately in the related question of whether a single particle can be entangled and whether this entanglement can be used in quantum information schemes [9–15]. In this paper we consider the specific case of a superposition of a single particle (atom or photon) and the vacuum, i.e.,

$$|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}, \tag{1}$$

and present the details of specific interferometric schemes that could be used to observe this superposition.

It is important to note that (1) is quite different from the state created by passing a single particle through a 50:50 beam splitter. A typical 50:50 beam splitter is a semi-reflective mirror that reflects half the incident light and transmits the rest. In general, this performs the following unitary transformations between the creation operators for particles at the input and output ports,

$$a^\dagger_{\rm out} = \left(a^\dagger_{\rm in} + ib^\dagger_{\rm in}\right)/\sqrt{2}, \tag{2}$$

$$b^\dagger_{\rm out} = \left(ia^\dagger_{\rm in} + b^\dagger_{\rm in}\right)/\sqrt{2}. \tag{3}$$

Using Eqs. (2), (3) we can calculate the transformation of any input to the beam splitter, for example the input $|1\rangle|0\rangle$ gives

$$|1\rangle|0\rangle \to (i|1\rangle|0\rangle + |0\rangle|1\rangle)/\sqrt{2}. \tag{4}$$

The most notable difference between this state and (1) is that the beam splitter conserves particle number whereas state (1) does not. Superpositions of the form

[1] The article is published in the original.

of (1) could be achieved by the action of a Hadamard gate (H) in the Fock basis $\{|0\rangle, |1\rangle\}$:

$$|0\rangle \xrightarrow{H} (|0\rangle + |1\rangle)/\sqrt{2}, \tag{5}$$

$$|1\rangle \xrightarrow{H} (|0\rangle - |1\rangle)/\sqrt{2}. \tag{6}$$

This is just a mathematical formalism and doesn't mean much unless we can find a way of implementing it. This paper will address precisely that issue: whether we can create a Hadamard gate that operates in the basis $\{|0\rangle, |1\rangle\}$.

QUANTUM STATE TRUNCATION

Before we discuss the Hadamard gate, it is worth reviewing a technique called quantum state truncation (QST) that will be important in our scheme. This was first put forward in 1998 [16] and involves creating truncated versions of quantum superpositions conditioned on particular measurement outcomes.

Suppose, for example, we had a coherent state with amplitude α and phase θ, i.e., $\left|\alpha e^{i\theta}\right\rangle_c$. This coherent state can be written as a superposition of Fock states.

$$\left|\alpha e^{i\theta}\right\rangle_c = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{(\alpha e^{i\theta})^n}{\sqrt{n!}}|n\rangle. \tag{7}$$

We will use a subscript c to denote coherent states; kets without a subscript are taken to be Fock states. Suppose, now, that we want to keep only the first two terms of the superposition, i.e., the ones corresponding to $|0\rangle$ and $|1\rangle$. We can achieve this by using the QST scheme shown in Fig. 1a, which works as follows. We input a single particle and a vacuum state into the two inputs of a 50:50 beam splitter (BS1 in Fig. 1a). One of the outputs is then combined with our coherent state at a second beam splitter (BS2). The outputs from BS2 are recorded at detectors A and B and if one particle is detected at A and none at B, then the remaining (and as yet unaccounted for) output from BSl is the truncated state that we wanted, i.e.,

$$|\psi_{\text{out}}\rangle = \left(|0\rangle + \alpha e^{i\theta}|1\rangle\right)\Big/\sqrt{|\alpha|^2 + 1}. \tag{8}$$

We can see this result by explicity calculating how the input state is transformed by the setup shown in Fig. 1a. The initial state is:

$$|\psi_{\text{in}}\rangle = \left|\alpha e^{i\theta}\right\rangle_c |1\rangle|0\rangle = \left(|0\rangle + \alpha e^{i\theta}|1\rangle + \cdots\right)|1\rangle|0\rangle, \tag{9}$$

where we have expanded the coherent state in the Fock basis and ignored any overall normalisation. The last two kets (qubits) are then transformed by BS1 to give

$$\left(|0\rangle + \alpha e^{i\theta}|1\rangle + \cdots\right)\left(i|1\rangle|0\rangle_{\text{out}} + |0\rangle|1\rangle_{\text{out}}\right), \tag{10}$$

where the second qubit is directed towards BS2, and the last qubit is directed towards the output mode.

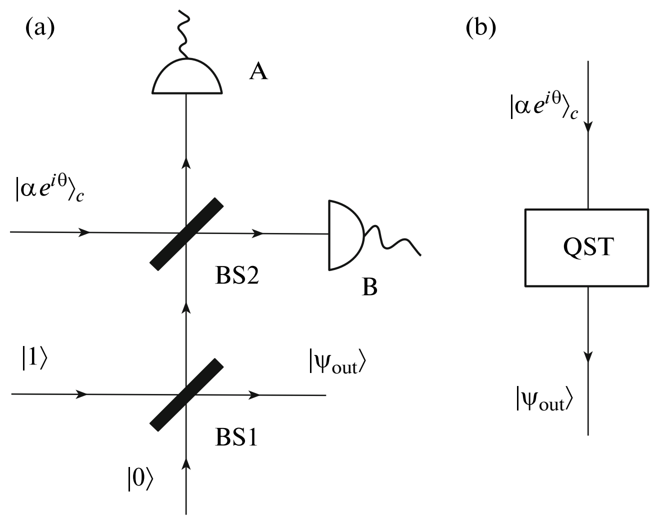

Fig. 1. Quantum state truncation scheme. (a) The Fock states $|0\rangle$ and $|1\rangle$ are fed into a 50:50 beam splitter (BS1) and one output is combined with the coherent state $\left|\alpha e^{i\theta}\right\rangle_c$ at a second 50:50 beam splitter (BS2). Depending on the measurement outcomes at detectors A and B, it is possible to achieve a truncated version of the coherent state at the output. For simplicity, the quantum state truncation scheme will be depicted as a "black box" (as shown in (b)) in the remainder of the paper.

Next, the first two qubits are transformed by BS2 to give the outputs at A and B. This gives,

$$\begin{aligned}&|0\rangle_A|0\rangle_B|1\rangle_{\text{out}} + |0\rangle_A|1\rangle_B\left(-|0\rangle_{\text{out}} + \alpha e^{i\theta}|1\rangle_{\text{out}}\right)\\ &\quad + i|1\rangle_A|0\rangle_B\left(|0\rangle_{\text{out}} + \alpha e^{i\theta}|1\rangle_{\text{out}}\right) + \ldots.\end{aligned} \tag{11}$$

From this, we see that if we detect 1 particle in A and none in B then the output state must be projected onto $|0\rangle + \alpha e^{i\theta}|1\rangle$, which, when normalised, is precisely the truncated state given by Eq. (8).

This QST procedure will be very useful when we consider a scheme for observing superpositions between a single particle and the vacuum. It is therefore convenient to represent it more compactly as a "black-box" as shown in Fig. 1b.

HADAMARD GATE

We now turn our attention to implementing a Hadamard gate in the Fock basis $\{|0\rangle, |1\rangle\}$. One possible scheme is shown in Fig. 2 and consists of three 50:50 beam splitters (labelled BS1, BS2, and BS3), two ordinary mirrors, a π phase shift, and two nonlinear crystals (labelled χ). We take the Hamiltonian for the nonlinearity to be

$$H_\chi = \hbar\chi a^{\dagger 2}a^2 = \hbar\chi\hat{n}(\hat{n} - 1), \tag{12}$$

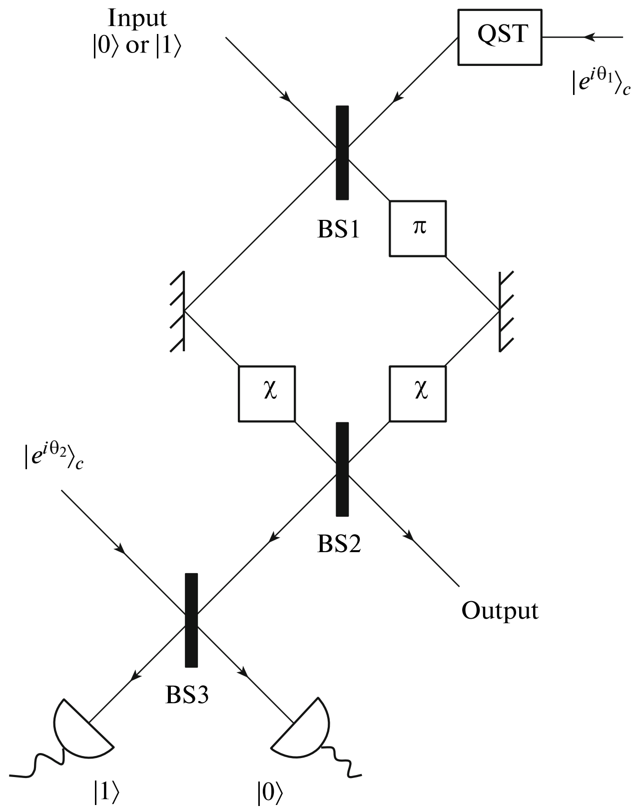

Fig. 2. The Hadamard gate scheme. This consists of three 50:50 beam splitters (BS1, BS2, and BS3), two nonlinearities (χ), a π phase shift (π) and conditional measurements made at the outputs of BS3. It also makes use of the quantum state truncation scheme shown in Fig. 1. The overall transformation that this interferometer performs is given by Eqs. (16) and (17).

where a is the annihilation operator corresponding to the mode on which the nonlinearity acts and $\tilde{n}$ is the corresponding number operator. We will take $\chi = \pi/2$, which means that there is no phase shift if there are 0 or 1 particles in the mode, but a phase shift of π if there are 2 particles. We can confirm that this setup performs a Hadamard operation by directly calculating how it transforms the input states.

Let us begin by considering the upper part of the setup consisting of the interferometer made up of BS1 and BS2. For now we take the input state to be $|0\rangle$ (we will consider $|1\rangle$ shortly). The other input to BS1 is given by performing QST on the coherent state $\left|e^{i\theta_1}\right\rangle_c$, which has an amplitude of $\alpha = 1$. We have seen that this gives $|0\rangle + e^{i\theta_1}|1\rangle$. So the overall input state to BS1 is $|0\rangle(|0\rangle + e^{i\theta_1}|1\rangle)$, where we are ignoring the normalisation, and will use the convention that the left hand ket represents the left hand path at each point in the scheme. We can now propagate this state through the setup:

$$\begin{aligned} |0\rangle(|0\rangle + e^{i\theta_1}|1\rangle) &\xrightarrow{\text{BS1}} |0\rangle|0\rangle \\ &+ e^{i\theta_1}(|1\rangle|0\rangle + i|0\rangle|1\rangle)/\sqrt{2} \\ &\xrightarrow{\pi,\text{mirrors}} |0\rangle|0\rangle + e^{i\theta_1}(i|1\rangle|0\rangle + |0\rangle|1\rangle)/\sqrt{2} \\ &\xrightarrow{\chi} |0\rangle|0\rangle + e^{i\theta_1}(i|1\rangle|0\rangle + |0\rangle|1\rangle)/\sqrt{2} \\ &\xrightarrow{\text{BS2}} |0\rangle(|0\rangle + ie^{i\theta_1}|1\rangle)_{\text{out}}. \end{aligned} \tag{13}$$

The mirrors give a $\pi/2$ phase change due to reflection and we see that the nonlinearity in this case has no effect since there is, at most, one particle on each path. Carrying out a similar analysis for the input $|1\rangle$ gives

$$|1\rangle(|0\rangle + e^{i\theta_1}|1\rangle) \to -|1\rangle(i|0\rangle + e^{i\theta_1}|1\rangle)_{\text{out}}. \tag{14}$$

These two transforms give us something close to what we want since we get orthogonal output states depending on the input state and these are equally weighted superpositions of $|0\rangle$ and $|1\rangle$. The problem is that these each of the output states of the second qubit are coupled to a different state of the first qubit. This wouldn't matter if we were only interested in using $|0\rangle$ or $|1\rangle$ at a time as our input. However, we want to be able to input arbitrary superpositions of these two states $c_0|0\rangle + c_1|1\rangle$, for which the output would be

$$\begin{aligned} c_0|0\rangle + c_1|1\rangle &\xrightarrow{H} c_0|0\rangle(|0\rangle + ie^{i\theta_1}|1\rangle) \\ &- c_1|1\rangle(i|0\rangle + e^{i\theta_1}|1\rangle), \end{aligned} \tag{15}$$

and in this case, our second qubit at the output path is entangled with the first qubit at the other output from BS2. If we trace over the first qubit, we get a mixed state of $|0\rangle$ and $|1\rangle$ as our output at the second qubit, which is not what we want. To avoid this, we need to wash out this which-way information by a nonunitary operation. We do this by combining the first qubit with another state at BS3. We then detect the number of particles at the outputs and retain only the cases where we detect one particle at the left hand detector and none at the right hand one. This is reminiscent of the QST scheme described above.

We can intuitively see how this works. If we detect only one particle at the two outputs of BS3 we do not know whether it came the left or right hand input. In other words, we don't know what the state of the qubit was that our output was coupled to. The detection irreversibly washes out this information. A straightforward calculation of this shows that the transformations brought about by the total scheme depicted in Fig. 2 is:

$$|0\rangle \to \frac{i}{\sqrt{2}}e^{i\theta_2}(|0\rangle + ie^{i\theta_1}|1\rangle)_{\text{out}}, \tag{16}$$

$$|1\rangle \to -\frac{1}{\sqrt{2}}(i|0\rangle + e^{i\theta_1}|1\rangle)_{\text{out}}. \tag{17}$$

This is equivalent to a Hadamard gate apart from some unimportant phases that could easily be removed to get (5) and (6).

Our scheme depends on post-selecting certain measurement outcomes. The probability that the Hadarmard is successful will depend on the success of the QST as well as the wash-out process. We have a probability of $1/(2e)$ for a quantum state truncation to succeed if it is fed with an equally weighted superposition of the vacuum and one photon state in the coherent state, i.e., for a $\left|e^{i\theta}\right\rangle_c$ state [16]. This is true for ideal photo detectors, nevertheless, even if we had photodetectors with efficiency of $\eta = 0.5$, we would still have a probability of $0.9/(2e)$. Likewise, the success probability for the wash-out stage can be shown to be $1/(2e)$ for ideal photodetectors. The overall probability of success for the Hadamard gate is therefore $(1/(2e))^2 \approx 0.034$. This is admittedly quite low. However, we really just want to demonstrate a proof of principle here. Also, there are ways to improve the success rate. For example, we could double the success probability at each QST stage if we didn't disregard the output state when we detected one particle in B and none in A in Fig. 1a and instead subjected it to a phase shift of π. A similar logic could be applied to the wash-out scheme at BS3, which would give us an overall success probability of about 0.14.

INTERFEROMETER

There is little point in creating a superposition of a particle and the vacuum if we cannot confirm that it has been created. The classic signature of a coherent superposition is an interference pattern. We could observe an interference pattern in this case by combining two Hadamard gates with a phase shift, ϕ, between them to create an interferometer as shown in Fig. 3a.

The problem with this approach is that it is complicated since each Hadamard is implemented by the scheme shown in Fig. 2. We can simplify things considerably if we consider only a particular input to our interferometer, let's say the vacuum, $|0\rangle$. In this case, the first Hadamard gate in Fig. 3a creates a superposition of a particle and the vacuum, but this could be achieved more easily using QST on a coherent state. Consequently, the input state and first Hadamard gate can be replaced by a coherent state and the QST gate as shown in Fig. 3b. Let us now consider how this works.

After the input state, $\left|e^{i\theta}\right\rangle_c$ has been truncated and had a phase shift, ϕ, applied to it (i.e., the first two steps in Fig. 3b), we end up with the state,

$$\left(|0\rangle + e^{i(\theta+\phi)}|1\rangle\right)/\sqrt{2}. \tag{18}$$

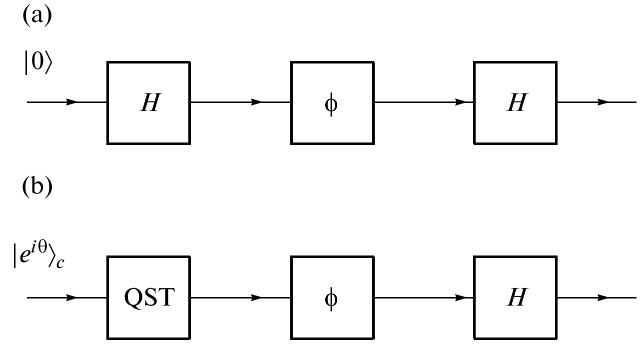

Fig. 3. (a) The general Hadamard interferometery scheme. An input state in the basis $\{|0\rangle, |1\rangle\}$ is transformed with a Hadamard gate, a phase shift, ϕ, and then a second Hadamard gate. (b) The scheme can be simplified if we are only interested in a specific case that can demonstrate the superposition of a single particle and the vacuum. In this case, quantum state truncation is used to give the superposition state (similar to the output of the first Hadamard in (a)). This state then undergoes a phase shift and a Hadamard transformation.

This is then passed through a Hadamard gate using the scheme shown in Fig. 2. We have seen that this gate performs the transformations given by Eq. (16) and Eq. (17) and so the final state is

$$\begin{aligned} &\frac{1}{\sqrt{2}}\left(|0\rangle + e^{i(\theta+\varphi)}|1\rangle\right) \\ &\xrightarrow{H} \sin\left(\frac{\phi+\theta-\theta_2}{2}\right)|0\rangle - e^{i\theta_1}\cos\left(\frac{\phi+\theta-\theta_2}{2}\right)|1\rangle, \end{aligned} \tag{19}$$

where we have ignored an unimportant overall phase. The probabilities of measuring one particle and none at the output of the interferometer are respectively,

$$P_1 = \cos^2\left(\frac{\phi+\theta-\theta_2}{2}\right), \tag{20}$$

$$P_0 = \sin^2\left(\frac{\phi+\theta-\theta_2}{2}\right). \tag{21}$$

This looks promising as we have interference fringes at the output that depend on the applied phase ϕ. However, the fringes also depend on θ and θ_2, which presents a problem. To see interference fringes, we need to repeat the experiment many times for each value of ϕ in order to find the values of P_0 and P_1. However the phases of coherent states that are independently produced are not fixed and vary randomly from run to run. This means that on the ensemble average we need to average over all phases θ and θ_2 and, consequently, the interference fringes will wash out.

We can overcome this problem by fixing their *relative* value, i.e., $\theta - \theta_2$ by creating them from the same source. In Fig. 4, we show how this could be achieved by introducing another beam splitter (BS4) to create

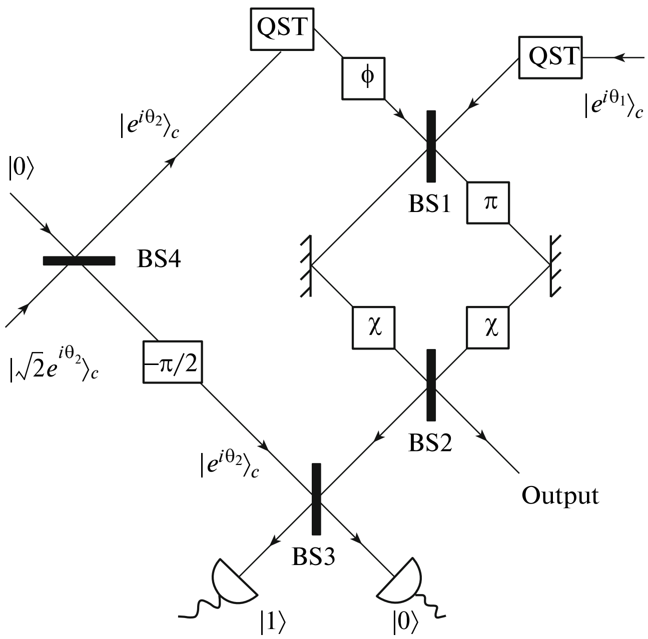

Fig. 4. The full scheme for demonstrating interference between a single particle and the vacuum as given in Fig. 3b. Importantly, as discussed in the text, the phase of the input to the QST scheme on the left and the input to the which-way wash-out scheme at BS3 have their phases fixed. This is achieved by creating them from a common source at BS4.

the states $\left|e^{i\theta}\right\rangle_c$ and $\left|e^{i\theta_2}\right\rangle_c$ so that their phase is the same on every run, i.e., $\theta - \theta_2 = 0$. In this case, the output probabilities are

$$P_1 = \cos^2(\phi/2), \tag{22}$$

$$P_0 = \sin^2(\phi/2). \tag{23}$$

This means that interference fringes should be able to be built up over an ensemble of runs since the position of the fringes now depends only on the controllable parameter, ϕ.

MIXED STATES

So far it seems that we have relied on the somewhat circular argument that we need to start with a superposition of number states (i.e., a coherent state) in order to observe a superposition of the form of (1). In this section, we show that we do not need to start with superpositions at all and can use mixed state inputs instead.

The mixed state,

$$\rho = e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}|n\rangle\langle n|, \tag{24}$$

is formally equivalent to a coherent state with amplitude $|\alpha|$ averaged over all phases, i.e.,

$$\rho = \frac{1}{2\pi}\int_0^{2\pi}\left||\alpha|e^{i\theta}\right\rangle\left\langle|\alpha|e^{i\theta}\right|d\theta. \tag{25}$$

This means that, if $\rho_M = \rho_1 \otimes \rho_2 \otimes \ldots$ is the input density matrix to our full interferometer shown in Fig. 4 with mixed states

$$\rho_1 = e^{-2}\sum_{n=0}^{\infty}\frac{2^n}{n!}|n\rangle\langle n|, \tag{26}$$

$$\rho_2 = e^{-1}\sum_{n=0}^{\infty}\frac{1}{n!}|n\rangle\langle n|, \tag{27}$$

instead of the coherent states $\left|\sqrt{2}e^{i\theta_2}\right\rangle_c$ and $\left|e^{i\theta_1}\right\rangle_c$, the output is just what we calculated earlier, but averaged over all phases θ_1 and θ_2. From (19), we see that we get

$$\rho = \sin^2(\phi/2)|0\rangle\langle 0| + \cos^2(\phi/2)|1\rangle\langle 1|, \tag{28}$$

where we have used the fact that relative values of θ and θ_2 are fixed, i.e., $\theta - \theta_2 = 0$ and we have averaged over θ_1 and θ_2. The output state is therefore a mixture of a single particle and the vacuum. Interestingly, the probabilities of detecting one or no particles at the output are not affected. We can argue that this is consistent with there being a superposition of a single particle and the vacuum inside the interferometer if we think more carefully about the state where the phase, φ, is applied. In order to do this, it is convenient to slightly modify the scheme shown in Fig. 4 by adding a QST step after the $-\pi/2$ phase shift on the lower output from BS4. It is easy to see that this doesn't affect the final output from the interferometer because it just throws away all cases where there were two or more particles on that path. However, the post-selection procedure at BS3 achieved the same effect.

If we consider the particular mixed state ρ_1 given by Eq. (26) as the input to the 50:50 beam splitter BS4, then the output state after the QST on each path is

$$\begin{aligned}\rho_1 \to (1/4)(&|0,0\rangle\langle 0,0| + |1,1\rangle\langle 1,1| + |0,1\rangle\langle 0,1| \\ &+ |1,0\rangle\langle 1,0| + |0,1\rangle\langle 1,0| + |1,0\rangle\langle 0,1|).\end{aligned} \tag{29}$$

We can see that this state is not entangled by taking the partial transpose and checking that there are no negative eigenvalues [17]. This means that the upper path after the QST operation is not entangled with the lower (reference) path after the QST operation.

This is important because it means that when the phase, φ, is applied to the upper path, it should depend only on the state on that path. However, we know that the output from the full scheme (28) depends coherently on the phase that is applied. This suggests that the state on the path where ϕ is applied is a superposition of number states. If we had a number state or a mixture of number states at that point then the phase would not alter the state, i.e.,

$$\rho = \sum_n P_n |n\rangle\langle n| \xrightarrow{\phi} \sum_n P_n e^{in\phi} |n\rangle\langle n| e^{-in\phi} = \rho, \quad (30)$$

where P_n are general probabilities. This seems strange: the total state (29) is mixed and yet the output from the scheme shown in Fig. 4 suggests that the upper path is in a superposition of a single particle and the vacuum. The answer is that the upper path is in a mixed state *on average* and we need to think about what happens run by run.

One way of interpreting these conclusions is that, on a given run, the output state of BS3 after the two QST operations is

$$\left(|0\rangle + e^{i\theta}|1\rangle\right)\left(|0\rangle + e^{i\theta}|1\rangle\right)/2, \quad (31)$$

where θ is a random phase and the left and right qubits respectively represent the upper and lower paths. On a single shot, we have a superposition of a single particle and the vacuum where the phase, ϕ, acts on the upper path. However, averaging over all phases gives us the mixed state (29). Normally, these two interpretations would be indistinguishable since an interference pattern needs to be built up over many detections and the fringes would wash out. However, we have arranged things so that the lower path *keeps track* of the random phase on each run. We can see this from (31) where the lower state has a record of the same random phase that appears in the upper path. This allows us to reconstruct the interference pattern indicating that we had a superposition of different Fock states inside the interferometer. Without the lower (reference) path no interference pattern would be seen.

A SIMPLER SCHEME

We can see a similar effect with a simpler (though perhaps less "clean") scheme that is worth discussing here. This just involves a straightforward modification of a Mach-Zehnder interferometer. The new setup is shown in Fig. 5. The Mach-Zehnder interferometer, composed of the two beam splitters BS1 and BS2, is modified by adding a QST operation to each path and also implementing a scheme to wash-out one of the output ports from BS2. The two inputs are the mixed states given by (26) and (27), i.e.,

$$\rho_1 = e^{-2}\sum_{n=0}^{\infty} \frac{2^n}{n!}|n\rangle\langle n|, \quad (32)$$

$$\rho_2 = e^{-1}\sum_{n=0}^{\infty} \frac{1}{n!}|n\rangle\langle n|. \quad (33)$$

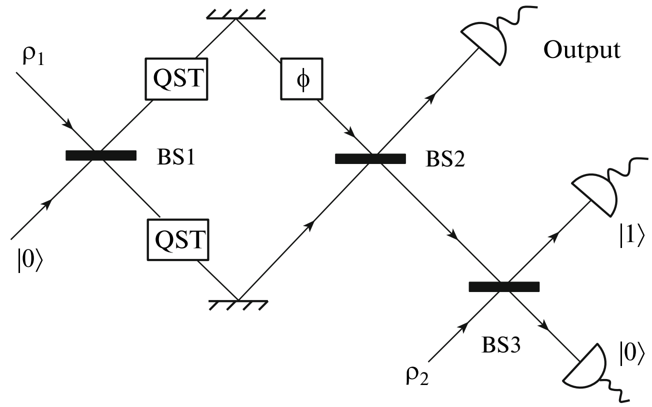

Fig. 5. The simpler scheme. A Mach-Zehnder interferometer is modified by putting a QST scheme on each path and by washing out the which-way information at the lower output port from BS2. Interference fringes that depend on the phase, ϕ, are seen in the number of particles detected at the port labelled "output."

Just after the two QST operations, the state inside the interferometer is

$$\begin{aligned}\rho_1 \to (1/4)(&|0,0\rangle\langle 0,0| + |1,1\rangle\langle 1,1| + |0,1\rangle\langle 0,1| \\ &+ |1,0\rangle\langle 1,0| - i|0,1\rangle\langle 1,0| + i|1,0\rangle\langle 0,1|).\end{aligned} \quad (34)$$

Apart from some phases, this is the same as (29). As before, we can rigorously show that there is no entanglement between the two paths of the interferometer. Finally, we need to wash out the which-way information at one of the output ports. This is important because if we know (or could know) the number of particles at both outputs, we will project the state inside the interferometer onto an entangled state, which is not what we want.

This wash-out scheme is implemented using BS3 and the mixed state ρ_2. Results are post-selected based on the measurement outcome of one particle at the upper output from BS3 and no particles at the lower output (see Fig. 5). As before, we can understand this process intuitively because it introduces an ambiguity as to whether the detected particle came from ρ_2 or from the output from BS2.

The state at the remaining output port (upper output from BS2) is then given by

$$\begin{aligned}\rho_{\text{out}} = (2/7)&\left[1 + 2\cos^2\phi/2\right]|0\rangle\langle 0| \\ &+ (4/7)\sin^2(\phi/2)|1\rangle\langle 1| + (1/7)|2\rangle\langle 2|.\end{aligned} \quad (35)$$

This clearly shows fringes that depend coherently on the value of ϕ, but is not as "clean" a result as the perfect fringes given by the previous scheme as shown in (28). For example, no fringes are seen for the case of two particles. This is not surprising since the only way that we could get two particles at the output is if each path inside the interferometer contained a single particle, i.e., the state was $|1\rangle|1\rangle$. This is insensitive to a phase shift on either path and so, not surprisingly, does not give an interference pattern that depends on ϕ. In the case that no particle is detected, the visibility of the fringes is 1/2. However, for a single particle, we get fringes with visibility one.

So, although the output state is not as clear as in the previous scheme, we still see fringes and the advantage is that the setup is much simpler. We can use the same argument as before that a consistent interpretation of these results is that the state on the path inside the interferometer where the phase acts must be a superposition of a single particle and the vacuum.

CONCLUSIONS

We have proposed two different ways of both creating and observing evidence for a superposition of a particle with the vacuum. Our schemes apply to both optical and atomic systems. They also begin with mixed states and so do not need to make any a priori assumptions about the existence of non-number-conserving superpositions. We should note that the output from both schemes is a mixed state and so they do not prepare superpositions that could, for example, then be used in other protocols. However, the probability of detecting one or no particles at the output depends coherently on a controllable external phase. This is consistent with the interpretation that there was a coherent superposition inside the interferometer at the point at which the phase, ϕ, was applied.

What we have presented is really a "proof of principle" that evidence consistent with non-number-conserving superpositions could be observed in low energy non-relativistic experiments. There may very well be simpler schemes for doing this. It is nonetheless a fascinating idea and it will be interesting to see how far its applicability can be extended to a broader range of physical systems or if there are fundamental limitations that prevent this.

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REFERENCES

1. G. C. Wick, A. S. Wightman, and E. P. Wigner, Phys. Rev. **88**, 101 (1952).
2. Y. Aharonov and L. Susskind, Phys. Rev. **155**, 1428 (1967).
3. R. Mirman, Phys. Rev. **186**, 1380 (1969).
4. A. Kitaev, D. Mayers, and J. Preskill, Phys. Rev. A **69**, 052326 (2004).
5. S. D. Bartlett, A. C. Doherty, R. W. Spekkens, and H. M. Wiseman, Phys. Rev. A **73**, 022311 (2006).
6. M. R. Dowling, S. D. Bartlett, T. Rudolph, and R. W. Spekkens, Phys. Rev. A **74**, 052113 (2006).
7. M. O. Terra Cunha, J. A. Dunningham, and V. Vedral, Proc. Roy. Soc. A **463**, 2277 (2007).
8. T. Paterek, P. Kurzynski, D. Oi, and D. Kaszlikowski, arXiv:quant-ph/1004.5184.
9. S. M. Tan, D. F. Walls, and M. J. Collett, Phys. Rev. Lett. **66**, 252 (1991).
10. L. Hardy, Phys. Rev. Lett. **73**, 2279 (1994).
11. E. Santos, Phys. Rev. Lett. **68**, 894 (1992).
12. S. M. Tan, D. F. Walls, and M. J. Collett, Phys. Rev. Lett. **68**, 895 (1992).
13. G. Bjork, P. Jonsson, and L. L. Sanchez-Soto, Phys. Rev. A **64**, 042106 (2001).
14. J. A. Dunningham and V. Vedral, Phys. Rev. Lett. **99**, 180404 (2007).
15. J. A. Dunningham, V. Palge, and V. Vedral, Phys. Rev. A **80**, 044302 (2009).
16. D. T. Pegg, L. S. Phillips, and S. M. Barnett, Phys. Rev. Lett. **81**, 1604 (1998).
17. M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Lett. A **223**, 1 (1996).