PROCEEDINGS OF THE ROYAL SOCIETY A

Proc. R. Soc. A (2007) **463**, 2277–2286
doi:10.1098/rspa.2007.1887
Published online 3 July 2007

Entanglement in single-particle systems

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We address some of the most commonly raised questions about entanglement, especially with regard to the so-called occupation number entanglement. To answer unambiguously whether entanglement can exist in a one-atom delocalized state, we propose an experiment capable of showing violations of Bell's inequality using only this state and local operations. We review previous discussions for one-photon non-locality and propose a specific experiment for creating one-atom entangled states, showing that the superselection rule of atom number can be overcome. As a by-product, this experiment suggests a means of creating an entangled state of two different chemical species. By comparison with a massless system, we argue that there should be no fundamental objection to such a superposition and its creation may be within reach of present technology.

Keywords: entanglement; field theory; modes; atom–molecule entangled state

1. What is entanglement?

Ever since entanglement was characterized as a resource, people have asked questions like 'is there entanglement in such a state/system?' or even more subtly 'how much entanglement is there in such a state/system?' In this paper, we want to emphasize that in isolation these questions are meaningless. They presuppose some important details which, if left unstated, can give rise to considerable misunderstanding.

Bohr (1935) pointed out that any discussion of quantum mechanics must be contextual, in the sense that a concrete physical situation should be described, including the measuring apparatus. In the same way, even the most abstract discussions of entanglement must clearly state which subsystems are being considered, since it is between these subsystems that entanglement may or may not appear. Identical states can exhibit entanglement between certain subsystems in one description and yet not in another. A good example is provided by the simple and well-studied system of a hydrogen atom (Cohen-Tannoudji *et al.* 1992). To diagonalize the hydrogen atom Hamiltonian, the first step is to change coordinates from proton and electron to the centre of mass and relative position of the particles. The centre of mass is free, and the problem is reduced to the relative separation, which is subject to a central potential. One then finds the eigenstates of the relative particle and, due to the

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Received 1 February 2007
Accepted 1 June 2007

10^3 ratio between the masses of proton and electron, it is a good approximation to call these the *electronic states* of the atom. Consequently, the groundstate of a hydrogen atom is a tensor product of the groundstate of the free centre of mass (i.e. a plane wave of zero momentum) and the groundstate of the relative particle (i.e. the spherical 1s orbital). There is no entanglement in this description. However, if one returns to a description in terms of the proton and the electron, the pure state that results is non-factorizable and hence entangled (Tommasini *et al.* 1998). It is true that the large mass asymmetry implies that the degree of entanglement will be small. Nevertheless, one could instead consider the positronium system in the same context and the result would be that the positron and the electron were as entangled as the particles in the original Einstein–Podolsky–Rosen state (Einstein *et al.* 1935).

Another physically appealing example of such a situation is a system of coupled harmonic oscillators. The first step to solve this problem is to transform to normal modes. In doing so, the eigenstates of the system are tensor products of the eigenstates of the normal modes and, as such, there is no entanglement at all between these modes. However, entanglement may be seen in a description of the original oscillators' modes. Similarly, the vibrational groundstate of a crystal lattice will be a direct product of groundstates of each of its phonon modes, but this same state shows entanglement among the constituent ionic cores (Ashcroft & Mermin 1976).

The essential point here is that entanglement is not an *absolute* property of quantum states. Entanglement is a property of a quantum state *relative* to a given set of subsystems. Even the quantum information community's favoured system of a pair of qubits has been shown to have an infinity of *tensor product structures* (Zanardi *et al.* 2004), which means that the same two-qubit state can be entangled or not, depending on the structure chosen. This last example illustrates another common misconception. The other three examples we gave were based on Hamiltonians that coupled the subsystems; however, one must avoid thinking that direct interaction is necessary for the appearance of entanglement (Pan *et al.* 1998). It is just one easy and natural way of (generically) creating entanglement among the subsystems.

In appendix A, we state and prove a theorem which says that, in a state space which can be written as a composite system (i.e. as a tensor product of other state spaces), any given pure state can be viewed as an unentangled state. If one also remembers that, in such a state space, a generic pure state is not factorizable, one can understand why a question like 'is there entanglement in such a state?' is meaningless in isolation.

In §2, we consider an even more controversial point (Wiseman & Vaccaro 2003; van Enk 2005): is it possible to have entanglement in a one-photon state?

2. Entanglement of particles versus entanglement of modes

The vast majority of discussions about entanglement start with a well-defined system of particles such as two photons, or three atoms, or one atom and one photon. At the same time, the quantum information community would like to consider entanglement to be a fundamental aspect of quantum mechanics that transcends any specific realization.

If we want to increase the breadth of applicability of entanglement, we should think in terms of fields which are a fundamental description of nature. Particles are only a manifestation of certain special configurations of quantum fields. If entanglement is to be considered a fundamental property of nature, and even a resource to be understood and applied, one would like to understand entangled fields.

To be more specific, let us discuss what happens to a photon when it reaches a beam splitter. If we consider just one spatial mode passing through the beam splitter and another reflected, this state, in occupation number notation, is

$$|10\rangle + |01\rangle, \tag{2.1}$$

which is just a Bell state for the two-mode system. Throughout this paper, we shall neglect unimportant normalizations. In this example, the two spatial modes are the relevant subsystems for considering entanglement. It is the state of the two spatial modes that is non-factorizable.

A rich debate has ensued over the consequences of entanglement in a one-photon state. A natural question is, 'can one make a Bell's inequality measurement in this system?' Tan *et al.* (1991) have pointed out that the answer is yes. Hardy (1994) made use of an analogous scheme to obtain contradictions with local realism without inequalities. However, their schemes use homodyne detection, and the measurements that violate Bell inequalities are correlations between two photons in coincidence detections. Greenberger *et al.* (1995) reinterpreted Hardy's proposal in terms of the measured photons with Feynman amplitudes for all alternatives. In this description, Hardy's proposal was transformed into a familiar two-particle test and so it was questioned whether this was truly a one-particle effect. Without really opposing the critics, Hardy replied (Hardy 1995) with a proposal for a very natural criterion for the existence of single-particle non-locality. The crucial test is whether the non-local effects in a system cease when the single-particle source is removed. However, adhering to such a criterion is a matter of taste and no final resolution could be gleaned from this debate. Curiously, Hardy points out that the same effect would not be possible for massive fields owing to superselection rules. Meanwhile, Greenberger *et al.* consider it to be a strength of their analysis that, by avoiding superpositions of the vacuum and one quantum, their interpretation could allow similar experiments with massive particles. Here, we not only affirm that single-photon entanglement exists, but we also provide an example of single-particle entanglement with massive fields (§4).

To begin with, let us use this one-photon entangled state (2.1) to locally create a two-particle entangled state which can be used to violate Bell inequalities, without involving extra photons. An 'easy' way is to resonantly couple each mode with a two-level atom for a time corresponding to a π pulse, i.e. the photon is absorbed and the corresponding atom excited. In this case, the mode state will always end up in the vacuum, while the atoms assume the Bell state

$$|\mathrm{eg}\rangle + |\mathrm{ge}\rangle. \tag{2.2}$$

This state is now well suited to carrying out experiments to test Bell's inequality. Furthermore, one can say that this 'transfer of state' is part of the detecting apparatus, and that the Bell's inequality experiment is performed directly on the two-mode one-photon system. A similar proposal was made by Aharanov & Vaidman (2000) some time ago.

It is worth noting that Lee & Kim (2000) have already proposed a different scheme for violating Bell inequalities with only single photons and also for teleportation of a part of an entangled state. In this sense, we conclude that another common fallacy is to say that 'entanglement is a property of many-particle systems'. It is, however, correct to say that 'entanglement is a property of composite systems', i.e. systems that have more than one subsystem.

3. Entanglement and locality

Another common misconception is to consider entanglement and non-locality as one and the same issue, since locality naturally implies position distinguishability, which is not a condition for entanglement. To clarify this point, we will describe a Bell measurement demonstrating entanglement, which cannot be considered to be a manifestation of non-locality. In fact, it is just another system described by the state (2.1).

The origin of this (incorrect) generalization is natural: non-locality is a non-classical manifestation of quantum systems, and a large class of applications of entanglement deals with spatially separated laboratories, usually occupied by Alice and Bob. In such contexts, it seems natural to discuss non-locality. This is the origin of the idea of *local operations*, and, generally, it is in this sense that the quantum information community talks about non-locality rather than the sense implied by Relativistic Theory.

Let us now consider a photon linearly polarized at 45°. In the occupation number notation with respect to horizontal and vertical polarization modes, such a photon is described by the state (2.1). Again, it is an entangled state of these two modes, and it is again a good example of how different descriptions of one and the same system can result in different answers to the question 'is it entangled?' This state is factorizable if one uses the modes with crossed polarizations, i.e. 45° and 135°.

This leads us to the question of whether we can make a Bell measurement for this system or, more generally, whether we can measure an entanglement witness. Again, the answer is yes. The simplest strategy involves using a polarizing beam splitter to spatially separate the two modes and then the previous procedure can be used. One can even say that the beam splitter creates non-locality, since the state after the beam splitter is the same as the one discussed in §2. However, as before, all this can be considered as part of the measuring apparatus and, in this case, we measure entanglement at just one position in space. In this context, it does not seem fair to talk about non-locality.

In this example, we said that the beam splitter generates non-locality, but in accordance with §1, one should not say that it creates entanglement. A beam splitter is an example of a linear mode converter. It can naturally be interpreted as a device that changes the tensor product structure. More specifically, it changes the modes that are measured into those that exhibit entanglement. It has been shown that it can even convert one kind of non-classicality (squeezed states) into another kind of non-classicality (entanglement) (Duan *et al.* 2000).

4. An experiment showing entanglement of massive fields

Up to this point, we have discussed entanglement of massless fields, involving one photon distributed over two modes that then become entangled. We would now like to consider a similar situation involving one atom distributed over two spatially distinct locations. Such a state must be entangled with respect to the usual local tensor product structure of particle location. In particular, we would like to consider the question: 'is there any superselection rule which prevents massive one-particle entanglement?'

The physical set-up begins with a single atom that is in a coherent superposition of being in two atomic traps, T_1 and T_2. This can be described by the state

$$|A0\rangle + |0A\rangle, \tag{4.1}$$

where A means that there is an atom in the trap and 0 means there is not. The ordering refers to traps 1 and 2. This state is a direct analogue of (2.1) and (2.2) and is readily formed by allowing tunnelling between the two traps.

As in the photonic case (figure 1), it is important that something else 'drags' this atom for us to be able to make direct measurements that confirm its entanglement. Two atomic beams, B_1 and B_2, must be focused on the traps with controlling parameters (such as classical magnetic fields which tune a Feshbach resonance; Cornish *et al.* 2000) chosen in such a way that, if there were an atom A in trap T_i, it would be captured by the beam. This would result in a molecule AB in beam i, while trap T_i would be left empty. For simplicity, let us consider just one atom, B_i, in each beam. The evolution due to the interaction between the beams and the trapped atom (similar to the π pulse in §2) can be described as

$$\{|A0\rangle + |0A\rangle\}|B_1, B_2\rangle \mapsto |00\rangle\{|B_1A, B_2\rangle + |B_1, B_2A\rangle\}. \tag{4.2}$$

So far, we have not specified the B_i atoms. They can be identical to each other or not. They can also be of the same element as atom A or not. For the detection mechanism we have devised, it will be important that the B_i atoms are not of the same element as A. However, this is not a fundamental problem.

At this stage of the scheme, we have an entangled state of two chemically distinct species: an atom and a molecule. Now, we would like to verify this entanglement, for example, by means of a witness such as a Bell inequality or by using it to perform a classically inadmissible operation such as teleportation. In the remainder of this analysis, we will neglect the state of traps T_1 and T_2 since they have been factored out from the flying modes.

One straightforward operation that we could perform on a state like

$$|B_1A, B_2\rangle + |B_1, B_2A\rangle \tag{4.3}$$

is simply to measure whether there is an atom or a molecule at each spatial position. A mass spectrometer could be used to achieve this. Another operation is to imprint a relative phase ϕ between the two possibilities, creating states of the form

$$|B_1A, B_2\rangle + e^{i\phi}|B_1, B_2A\rangle. \tag{4.4}$$

To achieve this, one needs only to apply a DC electric field to one of the spatial modes, since the Stark shift of the atom and the molecule should be different.

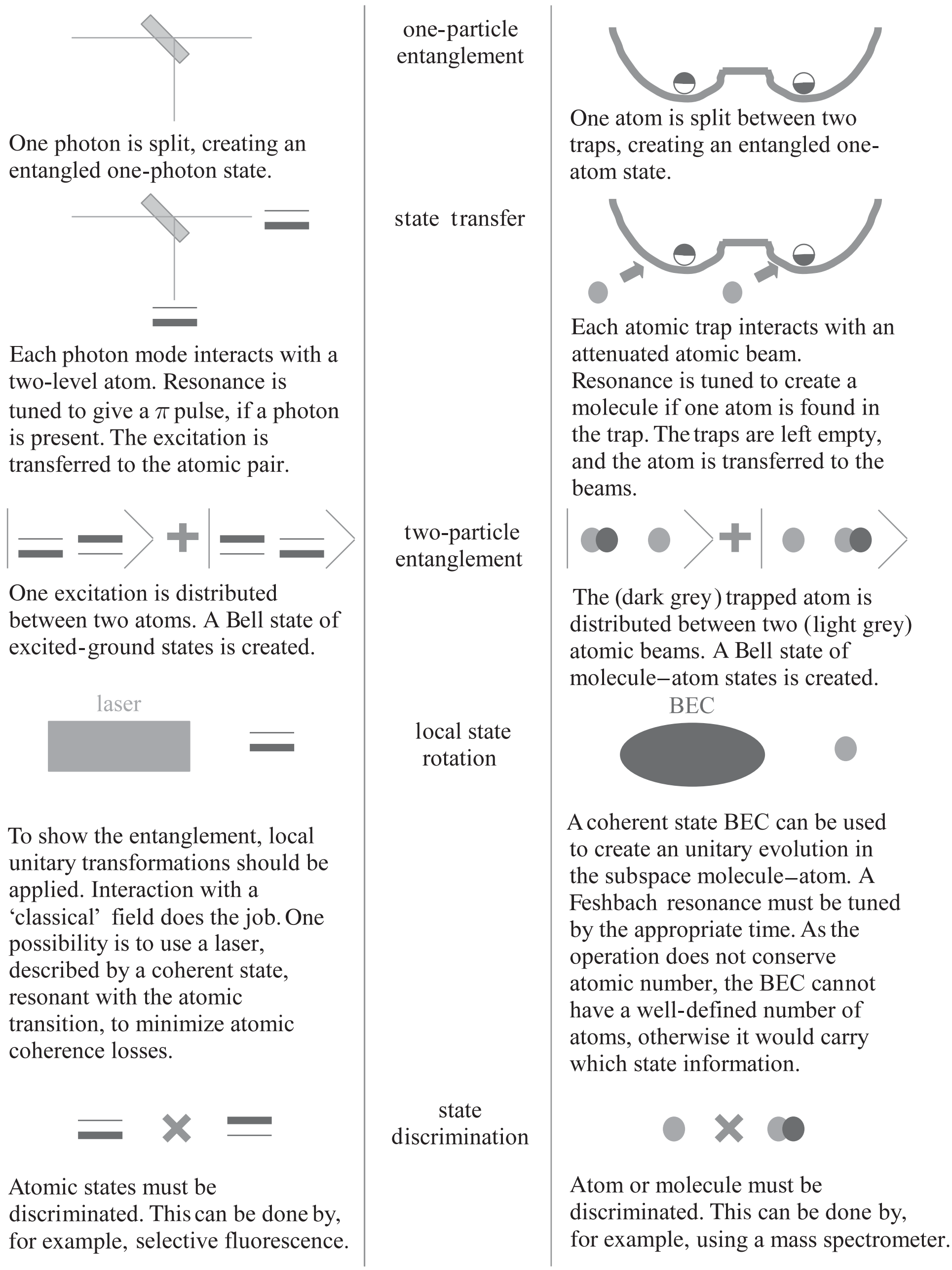

Figure 1. Schematic description of two experiments to find evidence for single-particle entanglement—one with a single photon, the other with a single atom. BEC, Bose–Einstein condensate.

However, these two operations alone do not test entanglement. In a qubit language, we are only measuring in the computational basis and applying a conditional phase shift to a state like $|\Psi_+\rangle$. We need to implement other local operations. To test a Bell inequality, we must be able to change basis. It is important to stress what *local* means here: it means that beams 1 and 2 must be addressed individually. The transformations that we need act in each beam separately and do the following:

$$|\mathrm{BA}\rangle \mapsto |\mathrm{BA}\rangle - |\mathrm{B}\rangle, \tag{4.5a}$$

$$|\mathrm{B}\rangle \mapsto |\mathrm{BA}\rangle + |\mathrm{B}\rangle. \tag{4.5b}$$

Clearly, such transformations do not conserve the number of A atoms locally, and cannot be implemented without extra A atoms. However, they can take place if we direct the beams onto other traps that contain a reservoir of A atoms. In order to discuss the situation, let us initially assume that we have one A atom in each of these additional traps. We shall lift this assumption later. We need to find a Feshbach resonance so that, in the presence of the appropriate field, the system evolves cyclically like a two-level system: $\mathrm{A}+\mathrm{B} \mapsto \mathrm{AB} \mapsto \mathrm{A}+\mathrm{B}$, with period t. If one waits only $t/4$, one will implement

$$|\mathrm{B},\mathrm{A}\rangle \mapsto |\mathrm{BA}\rangle + |\mathrm{B},\mathrm{A}\rangle, \tag{4.6a}$$

$$|\mathrm{BA},\mathrm{A}\rangle \mapsto |\mathrm{BA},\mathrm{A}\rangle - |\mathrm{B},\mathrm{A},\mathrm{A}\rangle. \tag{4.6b}$$

Such operation, followed by an atom-or-molecule detector, almost implements our 'reading in a different basis' scheme. However, the number of A atoms have 'which path' information, and, for this to work, it would be necessary to have a physical impossibility (and not a technical one) of knowing the number of A atoms remaining in the traps. Alternatively, one can say that for this situation there is a superselection rule for the number of A atoms (Verstraete & Cirac 2003), which must be circumvented for the implementation of the time evolution (4.5*a*) and (4.5*b*).

Despite these apparent problems, it can still be done. If we prepare a condensate in a state like a coherent state (which would also affect the interaction time, but this can be taken into account), then there would be essentially no difference between the original state in the trap and the state with one added atom. One should remember at this time that in order to apply a Ramsey pulse in the optical regime one usually uses a laser, and, if one tries to do this with a Fock state, it would simply not work since the atom and the field would become entangled, destroying the coherence of the atomic state. This optical analogy is emphasized in figure 1 and this massive Ramsey zone is well discussed in a recent work of Dowling *et al.* (2006).

Let us address with a little more care this Ramsey zone effect. In essence, what we need is that the reservoir which is used to 'rotate the state' of the system does not become correlated with it. In other words, it does not carry information about the quantum state. There are different ways of achieving this. In the original context of Ramsey interferometry, this corresponds to using a *classical* field resonantly coupled to the atomic transition $|\mathrm{g}\rangle \mapsto |\mathrm{e}\rangle$. One way of approaching classicality is to use a coherent state

$$|\alpha\rangle = \mathrm{e}^{\frac{-|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle.$$

In this case, the joint evolution takes the form

$$|\mathrm{e},\alpha\rangle \mapsto c|\mathrm{e},\alpha\rangle + s|\mathrm{g},\alpha_{\mathrm{g}}\rangle, \tag{4.7}$$

where c and s are amplitudes and $|\alpha_g\rangle$ denotes what can be called the coherent state with one added excitation. The important point here is that for $|\alpha| \gg 1$, $|\langle\alpha_g|\alpha\rangle| \to 1$, and the atomic state essentially factors out of the field. What is required in the present scheme for atoms and molecules is that our atomic reservoir in the trap can be described, in a second quantized notation, by $|\alpha\rangle$, where the Fock state $|n\rangle$ represents n atoms of element A. The coherence of Bose–Einstein condensates has been verified by Andrews *et al.* (1997).

After this change of basis, one needs only to discriminate between atom and molecule in each spatial mode to complete a Bell measurement for a one-atom entangled state. Of course, once again we are free to consider everything that happens after the state (4.3) has been created as just being part of the 'detector'. Furthermore, the apparent problem that the atom number is not conserved in the reservoir can be overcome by averaging the coherent state over all possible phases. This gives the density matrix for the reservoir as

$$\rho = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\theta |\alpha \mathrm{e}^{i\theta}\rangle\langle\alpha \mathrm{e}^{i\theta}|, \tag{4.8a}$$

which is equivalent to a mixed state with a classically unknown number of atoms

$$\rho = \sum_{n=0}^{\infty} \frac{\mathrm{e}^{-|\alpha|^2}|\alpha|^{2n}}{n!}|n\rangle\langle n|. \tag{4.8b}$$

5. Concluding remarks

In this paper, we have argued that the natural arena for discussing fundamental questions of entanglement is quantum field theory. Particles are effective concepts and effective theories are welcome whenever they can be applied. In this sense, the usual scenario of a well-defined number of multiple particles and subsystems is useful. However, there are certain contexts when this must be abandoned if we are to treat entanglement in a truly general sense. One consequence of this is the possibility of single-particle entanglement.

One-particle entanglement is as good as two-particle entanglement with respect to applications. A one-photon or one-atom state can be used to teleport a qubit, provided that it is delocalized (Lee & Kim 2000). Superselection rules for number of atoms are also effective concepts which can be overcome by appropriate schemes. Finally, we would like to emphasize that, despite both being manifestations of non-classicality, entanglement and non-locality are not synonyms.

The authors acknowledge a fruitful discussion on the theme with Prof. William Phillips. M.O.T.C. thanks Brazilian agencies CNPq, Fapemig and PRPq-UFMG for financial support. V.V. and J.A.D. thank the UK Engineering and Physical Sciences Research Council for support.

Appendix A. Statement and proof of a theorem

Theorem A.1. *Given a state vector* $|\psi\rangle$ *in a finite-dimensional state space* $\mathcal{H}$ *with non-prime dimension* $d = mn$, *there exists a tensor product structure* $\mathcal{H} \equiv \mathcal{V}^m \otimes \mathcal{W}^n$ *with respect to which* $|\psi\rangle$ *is factorizable.*

This theorem must be compared with two other results on linear algebra, with important applications on quantum mechanics. The first one is that, given any state vector, there exists an orthonormal basis including such a vector. It is indeed in the core of the proof of the above theorem. The other is the so-called Schmidt decomposition in which, given a state vector $|\psi\rangle$ and a bipartite tensor product structure $\mathcal{H} \equiv \mathcal{U} \otimes \mathcal{V}$, one can choose orthonormal basis $\{|u_i\rangle\}$ and $\{|v_j\rangle\}$ on the factors and write $|\psi\rangle = \sum_k \lambda_k |u_k\rangle \otimes |v_k\rangle$. The similarity is that one must remember that the theorem begins with the state $|\psi\rangle$ given. The tensor product structure will be adapted for this state.

Proof. One must remember that isomorphic vector spaces can be identified. Hence, just take two vector spaces $\mathcal{V}'$ and $\mathcal{W}'$ and choose orthonormal basis $\{|v_i\rangle\}$ and $\{|w_j\rangle\}$ for them. Choose an ordered orthonormal basis $\{|h_k\rangle\}$ for $\mathcal{H}$ with $|\psi\rangle$ as its first element. Set the isomorphism $\mathcal{V}' \otimes \mathcal{W}' \to \mathcal{H}$ given by $|v_i\rangle \otimes |w_j\rangle \mapsto |h_{(i-1)\times n+j}\rangle$. This isomorphism induces a tensor product structure in $\mathcal{H}$ and, with respect to it, $|\psi\rangle$ is factorized. ∎

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